ACRONYMS — THIS CHAPTER

ACRONYMS

BRDF	bidirectional reflectance distribution function
CC	CameraCalibration matrix (in the DNG pipeline); Creative Commons in a licensing context
CCD	charge-coupled device
CIE	Commission Internationale de l'Éclairage (International Commission on Illumination)
CIELAB	CIE Lab color space (lightness L with a, b color-opponent axes)
CMOS	complementary metal-oxide-semiconductor
D50	CIE standard illuminant D50 (~5000 K daylight)
D65	CIE standard illuminant D65 (~6500 K daylight)
DNG	Digital Negative (raw file format)
HDR	high dynamic range
HLG	hybrid log-gamma
ICC	International Color Consortium
ISP	image signal processor
PQ	perceptual quantizer (HDR transfer function)
SDR	standard dynamic range
SNR	signal-to-noise ratio
UCS	uniform color space

Course notes · 37 sections from the book · 14 optional

hide optional sections

the full book ↗

LECTURE NOTES 4 — COLOR

Course notes cut from *Computational Photography, an AI-powered Slopendium* for this lecture. Each section links back to its home in the book; sections marked **optional** go beyond what the lecture needs.

▸ Contents · 32 sections on this page, from 9 chapters of the book

Light and optics

Light: rays, waves, and the spectrum

OPTIONAL

in the book ↗

There is no single right answer to "what is light." Physics offers three models (Figure 2.1.1), and the trick is to pick the one that makes the question at hand easy. When we want to know *where* light goes, through a pinhole, a lens, or off a mirror, we treat it as a ray, a straight line obeying simple geometry. When we want to understand *color*, *diffraction*, or *interference*, we treat it as a wave, a periodic disturbance with a wavelength. And when we want to count light precisely, which is what ultimately limits image quality, we treat it as a stream of photons, discrete particles whose random arrival creates noise. The three pictures describe different aspects or approximations of the same physics.

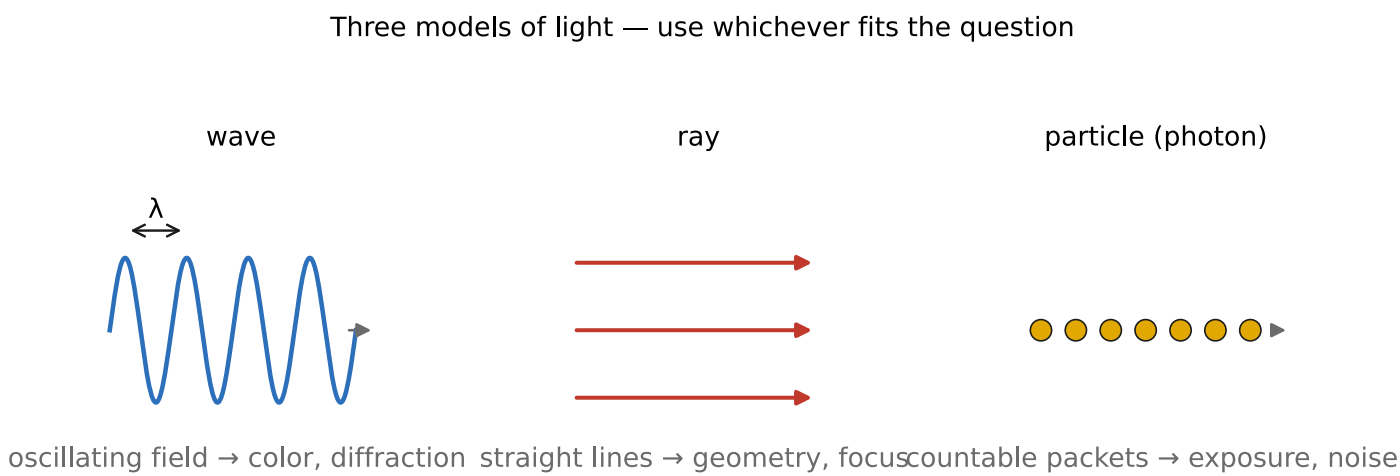


Figure 2.1.1 (reproduced). *Three models of light. The same beam drawn three ways: as a ray (a straight geometric line, for tracing paths through lenses and pinholes), as a wave (a periodic oscillation with wavelength λ, for color, diffraction, and interference), and as a stream of particles (discrete photons, for counting light and understanding noise). Each model answers a different class of question; we use whichever fits.*

The wave picture and the spectrum. Light is an electromagnetic wave, a coupled oscillation of electric and magnetic fields that travels through vacuum at a fixed speed $c \approx 3 \times 10^8$ m/s (about 300,000 km/s, and slower by a factor $1/n$ inside a medium of refractive index n). A single-frequency, or *monochromatic*, wave is characterized by one wavelength λ, the distance between successive crests, or equivalently by its frequency ν, the two tied by

$$c = \lambda\nu. \tag{1}$$

Visible light occupies a narrow band, roughly 400 nm (violet) to 700 nm (red), a small part of the electromagnetic (EM) spectrum that also includes radio, microwave, infrared, ultraviolet, X-rays, and gamma rays, all the same physics, differing only in wavelength (Figure 2.1.2). "Visible" is a fact about our eyes, not about our cameras: a standard silicon sensor responds to wavelengths the eye cannot see, into the near infrared beyond 700 nm and, with the right optics, into the ultraviolet below 400 nm, and computational photography can put that extra reach to work (Sensors).

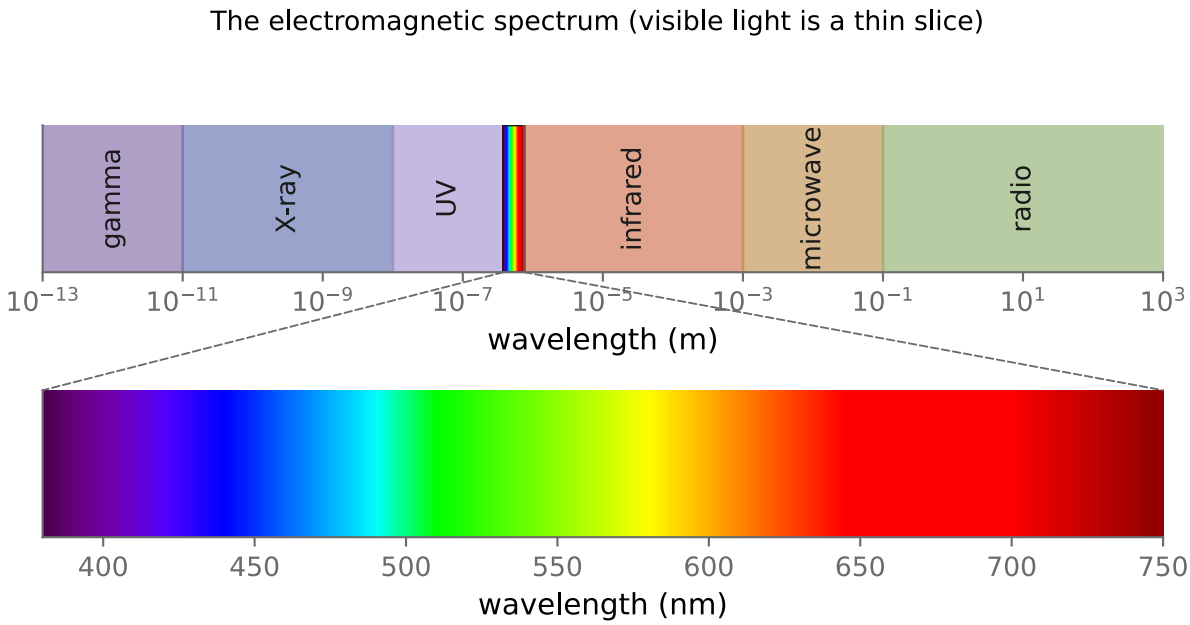


Figure 2.1.2 (reproduced). *The electromagnetic spectrum and the visible window. Wavelength runs from kilometer-scale radio waves to picometer gamma rays; the eye responds only to the narrow band from about 400 nm (violet) to 700 nm (red), shown expanded with its colors. Frequency and wavelength are linked by $c = \lambda\nu$. Photography lives almost entirely in this thin visible slice (with occasional excursions into the near-infrared and ultraviolet).*

Sometimes I try to picture what everything would look like if the whole spectrum were compressed into the visible spectrum. Also sometimes I try to picture your sister naked.

Figure 2.1.3 (reproduced). xkcd #273 "Electromagnetic Spectrum", Randall Munroe, xkcd.com/273 (CC BY-NC 2.5).

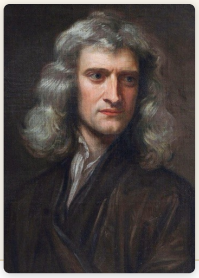
Most light is not monochromatic, however. A general wave is a superposition of many wavelengths, and what characterizes it is its *spectrum*: the distribution of energy over wavelength. That is, each wavelength has a given amount of energy, and this distribution characterizes the *physical color* of light. Physical color is different from the notion of color as perceived by humans, and the link between the two is fundamental in imaging; it is developed in Chapter 3.2, Perceptual color and trichromatic vision.

Newton's prism. Newton demonstrated the decomposition of white light into its spectrum. Pass a beam of white sunlight through a glass prism and it fans out into the full spectrum: the rainbow band of pure colors. The lesson is that white light is not a pure thing being "stained" by the glass; it is already a mixture of all wavelengths, and the prism merely bends each wavelength by a different amount, spreading the mixture out. Newton proved this with his *experimentum crucis* (Newton 1704 (Opticks)): he isolated one pure color from the spread and sent it through a second prism, and it did *not* split further. The pure colors are the irreducible ingredients; white is their sum (Figure 2.1.4).

Sidebar 1: who was Newton?

Isaac Newton (1642–1727) needs little introduction, but the prism is only the start of his presence in this book. The spectrum he split from white light is the physical basis of all the color science to come, and his *Opticks* established that color lives in the light, not the object. His corpuscular view of light competed for a century with the wave picture: both, we now know, are needed. On the

mathematics side we still run Newton's method (step by the inverse curvature, $H^{-1}\nabla f$) whenever a solver wants second-order convergence, and the Gauss–Newton refinement sits at the heart of feature tracking and bundle adjustment. *Portrait: James Thornhill after Godfrey Kneller, 1689, public domain (via Wikimedia Commons).*



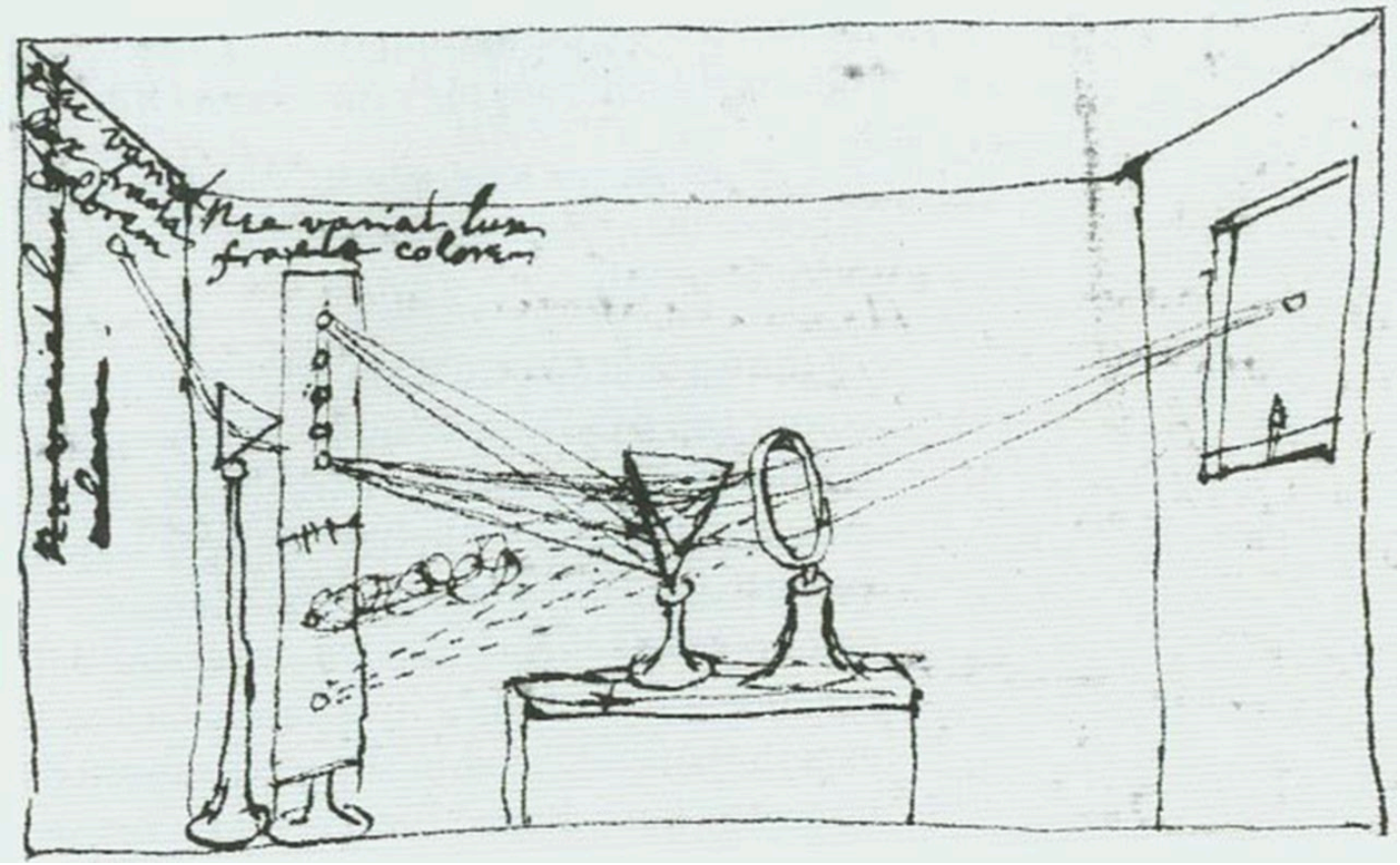


Figure 2.1.4 (reproduced). *Newton's prism experiment. White light entering a glass prism is dispersed into the spectrum because the prism bends each wavelength differently. The classic diagram: a second prism placed in the path of a single isolated color does not split it further, the experimentum crucis showing that white light is a mixture of pure wavelengths, not a uniform substance the prism colors.*

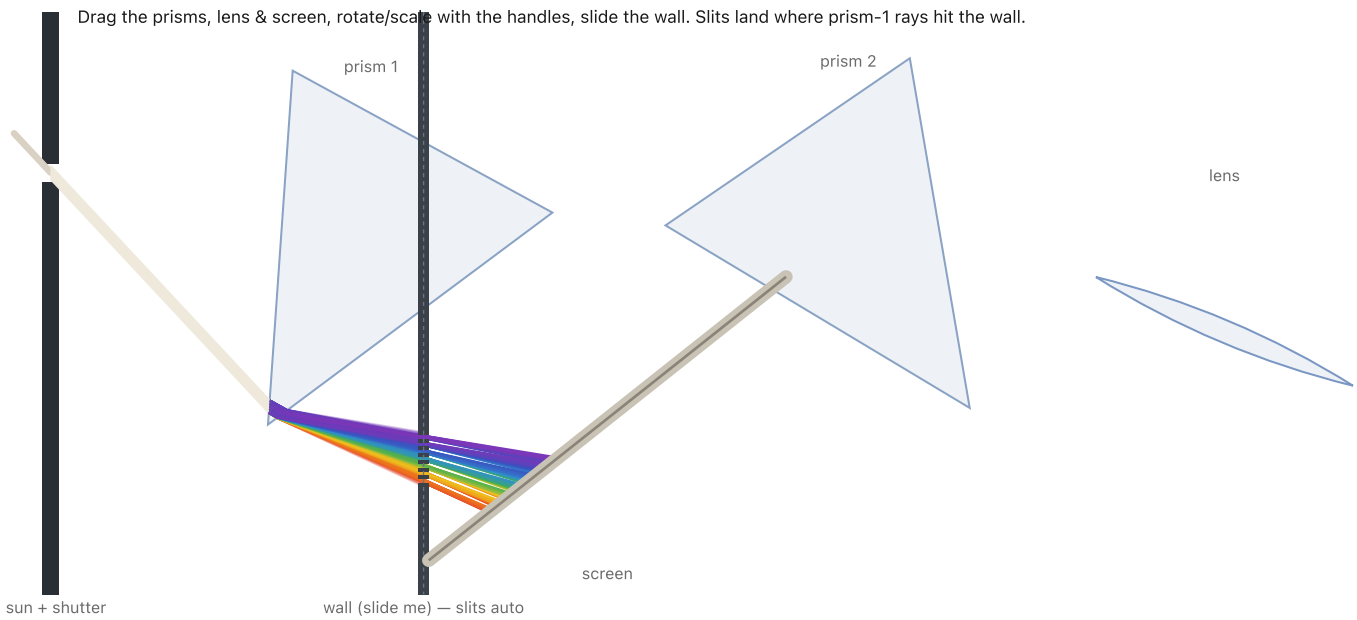


Figure 2.1.5 (reproduced). *The same experiment, rebuilt as a puzzle. Drag the two prisms and the lens, rotate and scale the selected piece with its handles, and slide the middle wall; the wall's slits appear automatically wherever the first prism's rays strike it, so you line the wall up with the spectrum rather than placing slits by hand. Set the prisms' dispersion and the lens's focal length; a hole-size slider widens the slits. Press Restart to scatter the setup, then Solution to snap it back to a working arrangement, or Show Newton's original to compare against Newton's own drawing of the experiment. Threading the spectrum through the slits, through the second prism, and onto the screen is a hands-on way to feel how dispersion, refraction, and focusing chain together: every ray keeps its single color through the second prism, the crux of Newton's* experimentum crucis.

So a spectrum is simply a function: power plotted against wavelength, $S(\lambda)$. You can think of it as an infinity of channels, one per wavelength.

💡 **BIG LESSON (L2.1) — LIGHT IS A SPECTRAL DISTRIBUTION**

Light, and the physical color it carries, is described by a spectral distribution: the power it carries at each wavelength, a function $S(\lambda)$. One wavelength is a single pure color; real light is a weighted mix of all of them. Everything downstream (how a prism fans light out, how a surface reflects it, how a sensor or an eye turns it into a perceived color) starts from this per-wavelength power.

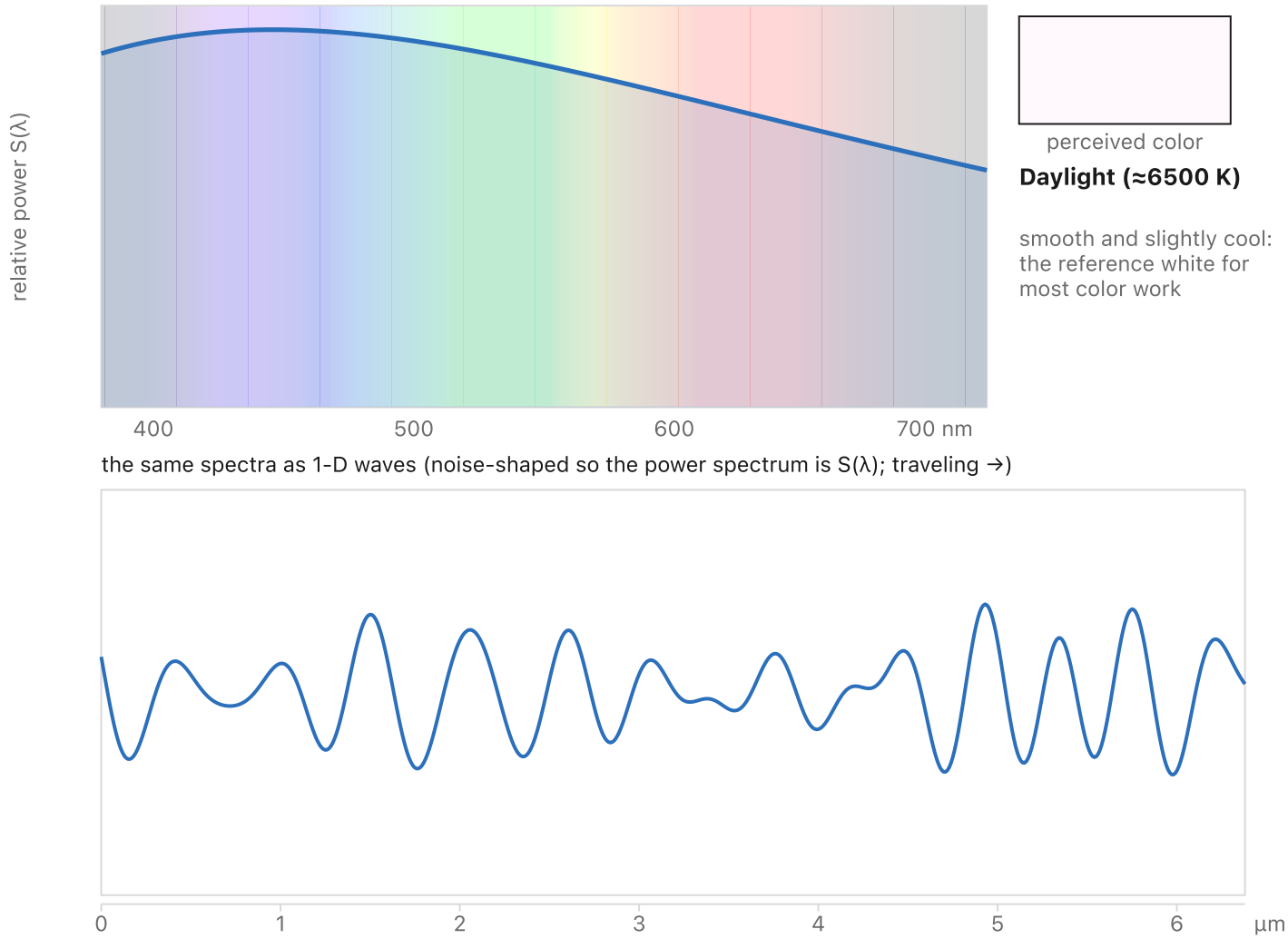


Figure 2.1.6 (reproduced). *The spectra of real light sources.* Each "white" source carries a different spectral power distribution $S(\lambda)$: daylight and tungsten are smooth Planck curves (cool and warm), a fluorescent tube adds sharp mercury lines on broad phosphor humps, a white LED is a blue spike feeding a yellow phosphor, and clear sky is daylight steepened by Rayleigh scattering. The swatch gives each source's perceived color; several read as nearly white despite very different spectra. *Interactive: pick a source, or overlay them all; each spectrum also plays as a 1-D traveling wave, and a bandpass-filter slider isolates a band to show the surviving wave and its color: a narrow band gives a clean tone and a saturated hue, a wide band stays noisy and washes toward white.*

Spectra come in two broad flavors. A *smooth* spectrum varies gently with wavelength: daylight, an incandescent bulb, the glow of any hot body. A *spiky* spectrum piles its power into a few narrow lines with little in between: a fluorescent tube (sharp mercury lines on a faint phosphor background), many cheap LEDs, a sodium street lamp, a laser. As we will discuss, spiky illuminants tend to be bad for photography because they single out a few wavelengths: their color rendering is poor (skin tones and saturated fabrics shift unpredictably), they defeat a single white-balance correction, and a scene under them can look sickly even though a meter calls the light white. Smooth, broadband light (daylight, tungsten, good full-spectrum LEDs) is forgiving; spiky light is treacherous. We return to this with white balance, color rendering, and the design of a camera's color filters.

Rays. The most useful everyday picture ignores the oscillation entirely and treats light as *rays*: straight lines along which light carries its energy. In a uniform medium a ray travels perfectly straight and changes direction only when it meets a surface, bending by refraction as it passes into glass or water and bouncing by reflection off a mirror. This is geometry rather than wave mechanics: to find where an image forms, you follow rays through the optics and see where they cross. It is a simplification, valid when the wavelength is tiny compared with the apertures and features the light meets, so that diffraction and interference are negligible, but it is the workhorse of this book. The overwhelming majority of what follows, image formation, focus, depth of field, exposure, nearly all of the optics, runs on ray optics, and often on an even simpler version of it, the paraxial thin-lens approximation. We reach for the wave picture only where it is unavoidable, for diffraction, interference, and the resolution limit; the rest of the time, light is rays.

Photons. The third picture is granular: light arrives in discrete packets called *photons*, and a detector, a camera pixel or a cell in the eye, absorbs them one whole photon at a time. Wave and photon are the same light seen two ways. The wave tells you where the light goes and how bright it is, and its intensity (the square of the amplitude) is really the density of photon arrivals; the photon is what actually gets counted. Each photon's energy is set by the wave's frequency, $E = h\nu$ (with h Planck's constant), so frequency is color: a blue photon (high frequency, short wavelength) carries more energy than a red one. We are simplifying here by giving each photon a single frequency, one pure wavelength; a real photon is a short wave packet with a small spread of wavelengths, but the one-frequency-per-photon picture is all photography needs. In this view the oscillation is not something a lone photon waves about in flight, it is the wave property that fixes the photon's color and energy, while the wave's amplitude sets how many photons show up. And because photons arrive at random, counting a finite number of them is inherently noisy, which is the shot noise that ultimately limits how clean an image can be.

The color of objects: illumination times reflectance

in the book ↗

Most ordinary photographed objects emit negligible visible light of their own; they only *reflect* whatever light falls on them. So the color that reaches your eye is the result of two spectra multiplied together, wavelength by wavelength:

$$C(\lambda) = S(\lambda)\,\rho(\lambda). \tag{2}$$

Here $S(\lambda)$ is the illuminant (the spectrum of the light source actually present) and $\rho(\lambda)$ is the surface's spectral reflectance, the fraction of light it sends back at each wavelength. Reflectance is the object's intrinsic, fixed signature: the apple's $\rho(\lambda)$ is high in the reds and low in the greens and blues, whatever the lighting. Illumination is whatever happens to be shining. Their per-wavelength product is what leaves the surface and reaches the eye (Figure 2.1.28). A red apple under white light reflects mostly red; the same apple under pure green light has almost nothing to reflect and looks nearly black, because there is no red illumination for its reflectance to act on.

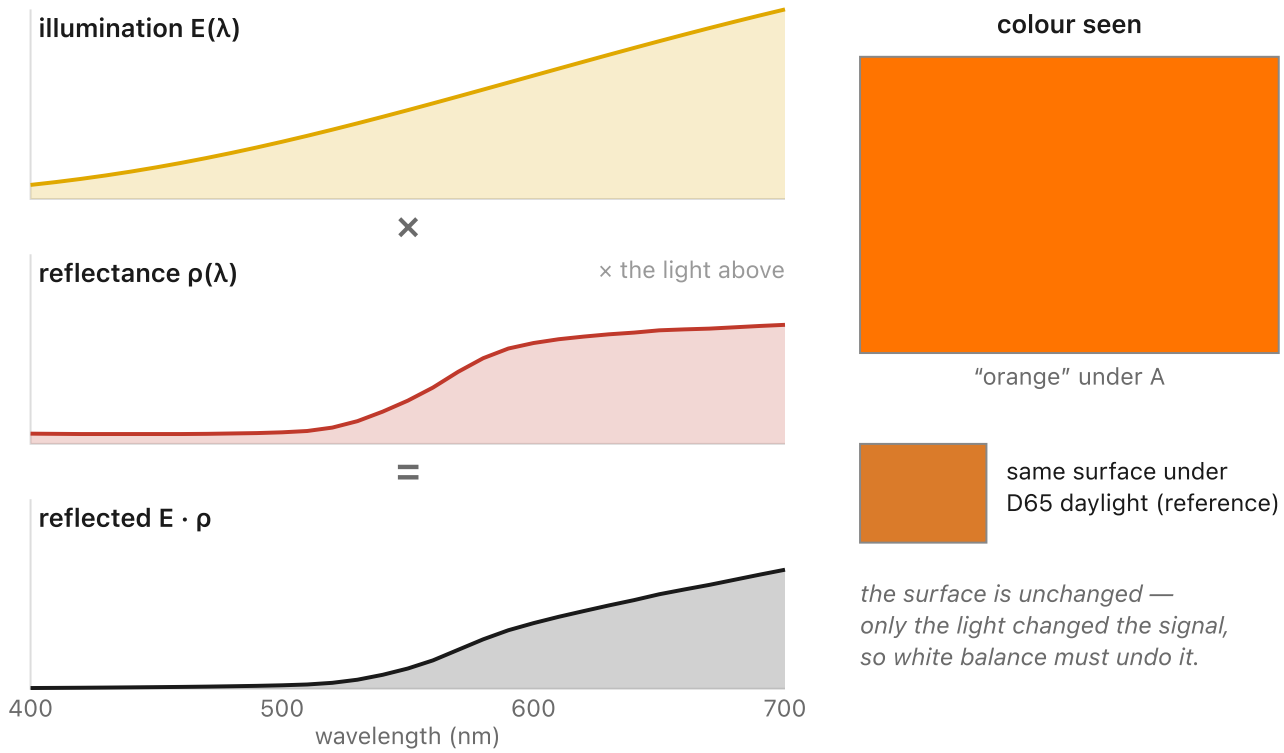


Figure 2.1.28 (reproduced). *Color is illumination × reflectance. A surface's fixed reflectance spectrum ρ(λ) (here an apple's, high in the reds) is multiplied per wavelength by the illuminant spectrum S(λ) actually present. The product C(λ) = S(λ)·ρ(λ) is the light that reaches the eye. Change the illuminant and the product changes even though the surface has not: the same reflectance under white versus reddish versus greenish light yields three different stimuli. One swatch shows the surface's unadapted appearance under the chosen illuminant; a smaller reference swatch shows it under D65 for comparison.*

This multiplicative entanglement is consequential, and it is the first instance of a principle that runs through the whole book.

💡 **BIG LESSON (L2.6) — THE MULTIPLICATIVE WORLD**

Much of imaging is *multiplicative*, not additive. The light reaching the eye is illumination **times** reflectance; contrast is a **ratio** of intensities; a shadow darkens a region by a **factor**, not by subtracting a constant. Because the structure is multiplicative, the natural tools are **multiply, divide, and log**, not just add. This is why, again and again, we will work with ratios and with logarithmic encodings rather than raw linear values. It is also why **white balance** is hard: to recover an object's true color we must *divide out* the unknown illuminant $S(\lambda)$ from the product $C(\lambda)$, undoing a multiplication we never observed directly. (The additive counterpoint, light from different sources *adding*, and what it means for choosing an encoding, comes in Color technology.)

Why this multiplicative structure matters so much is a question the rest of the book keeps answering. Our visual system does not care about the illumination for its own sake. It cares about *objects*, about the reflectance $\rho(\lambda)$ that tells it what a thing is and whether it is food, a face, or a threat. So the brain works hard to discount the illuminant, factoring the product $C(\lambda) = S(\lambda)\,\rho(\lambda)$ back apart to recover the reflectance and hold an object's apparent color and lightness steady as the light changes. That discounting is the perceptual machinery of lightness and color constancy, studied in *Human vision and color*, and a camera has to imitate it deliberately, which is exactly what white balance does. Much of imaging is this running effort to see past the illumination to the reflectance underneath.

Cone photoreceptors and perceptual color

in the book ↗

Even though cones are known to be the color vision receptors, a single cone does not measure wavelength; it produces one number, and stimuli of different wavelengths can yield the same response.

A given cone type k has a spectral sensitivity $c_k(\lambda)$: a curve saying how strongly it responds to light of each wavelength (Figure 3.2.1).

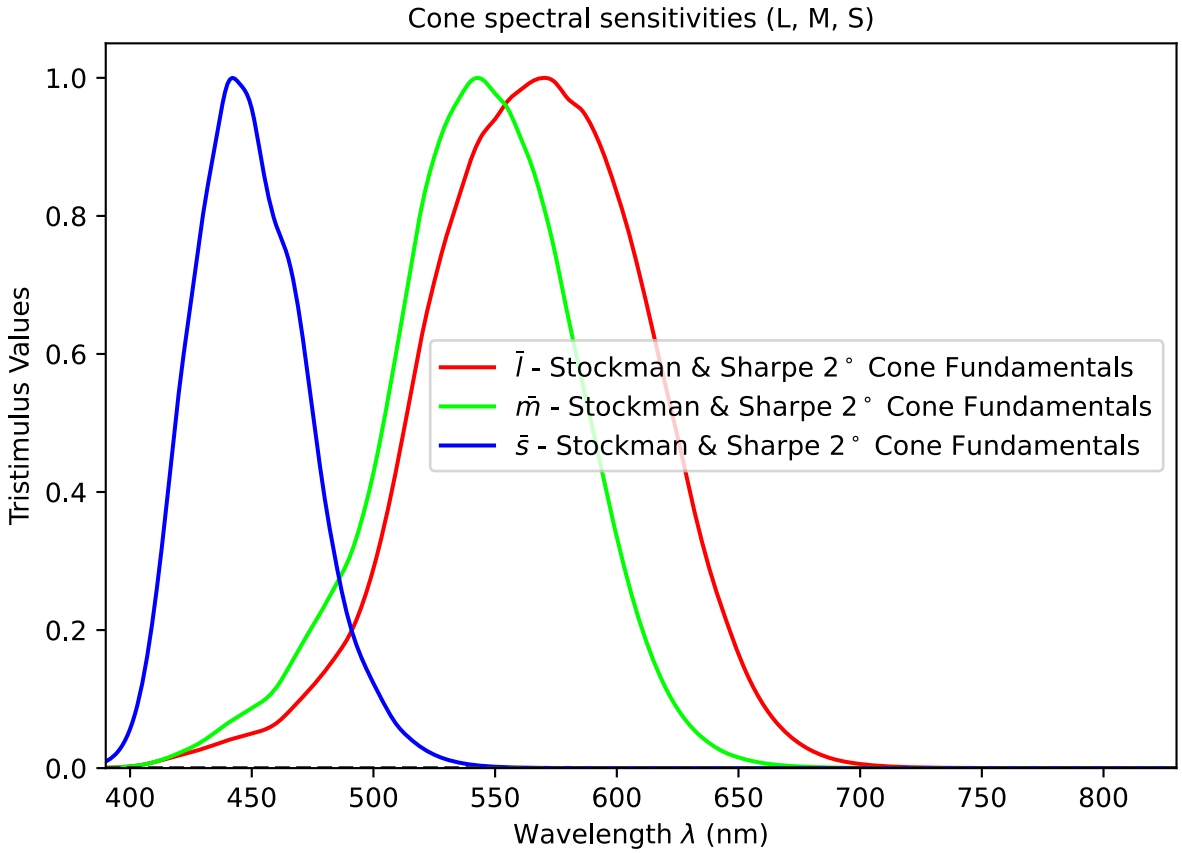


Figure 3.2.1 (reproduced). *The three cone spectral sensitivities. The long- (L), medium- (M), and short-wavelength (S) cones peak in the yellow-green, green, and blue-violet respectively. Note how heavily L and M overlap; they are far from independent "red" and "green" detectors. Each cone reports a single number: its sensitivity curve multiplied by the incoming spectrum and integrated.*

For a fixed state of adaptation, we model the initial cone excitation from an energy spectrum $E(\lambda)$ as a weighted integral, the spectrum weighted by its sensitivity and summed over all wavelengths (Figure 3.2.2):

$$r_k = \int E(\lambda)\, c_k(\lambda)\, d\lambda, \qquad k \in \{L, M, S\}. \tag{4}$$

This linear excitation model precedes the nonlinear adaptation and neural response. Use energy-based sensitivities with an energy spectrum, or quantum-based sensitivities with photon counts; mixing the two conventions gives a wavelength-dependent error. Mathematically the integral is a continuous dot product. In the case of monochromatic light (a single wavelength λ) it reduces to the scalar product of the cone's sensitivity at λ and the light's intensity for λ.

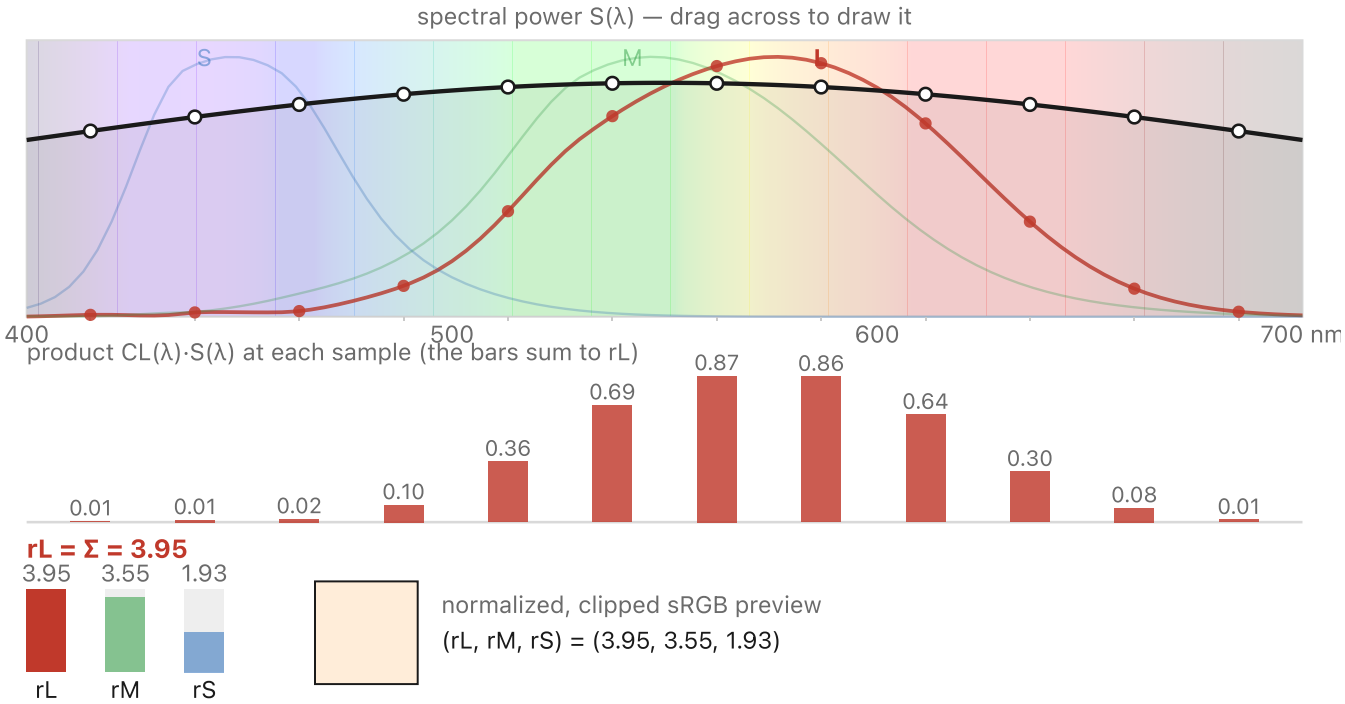


Figure 3.2.2 (reproduced). *From a spectrum to three numbers. Any incoming spectral power distribution is projected onto the three cone sensitivity curves (L, M, S) by a weighted sum over wavelength, collapsing an effectively infinite-dimensional spectrum into just three responses. Scribble a spectrum with the mouse (or load a real one, such as daylight or tungsten); at each sample wavelength the figure prints the spectrum value and the chosen cone's sensitivity, then writes out the full product-sum that adds up to that cone's response. This projection is exactly why different spectra can look identical (metamerism, next).*

As Figure 3.2.3 shows, lights at different wavelengths can yield the same response if their intensity ratio is the inverse of the cone's spectral response ratio for those two wavelengths. A single cone conflates wavelength with intensity, a property called *univariance*.

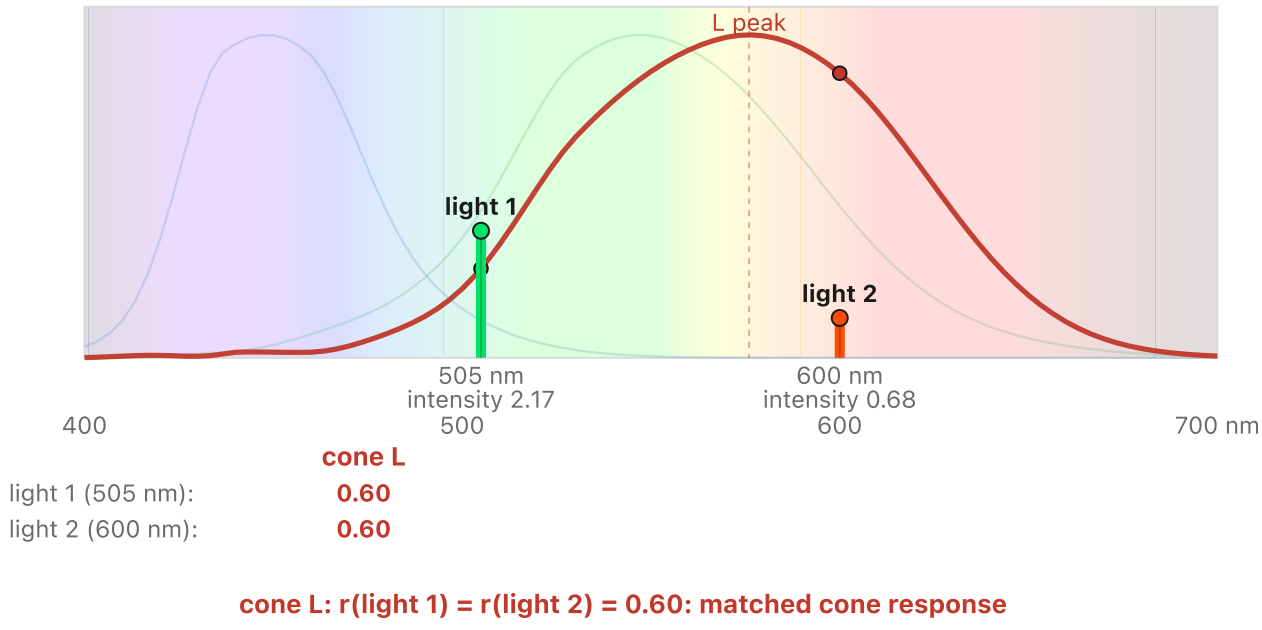


Figure 3.2.3 (reproduced). *A single cone cannot distinguish color. One cone type is chosen and its sensitivity is drawn bold, the other two faint. Two monochromatic lights sit at different wavelengths, and each light's intensity is set so the chosen cone gives the same response to both, so that cone alone cannot tell them apart. The other two cones respond differently, which is exactly the comparison the visual system uses to recover wavelength. This is univariance: for one cone, many wavelengths map to one output. Tick "show other cones" to reveal how the other two respond differently to the same two lights.*

While a single cone cannot distinguish wavelength, the combination of three cone responses provides much more information (click on "show other cones" in Figure 3.2.3). This is the *trichromatic* theory of color vision, pioneered by Thomas Young and put on a quantitative footing by Hermann von Helmholtz.

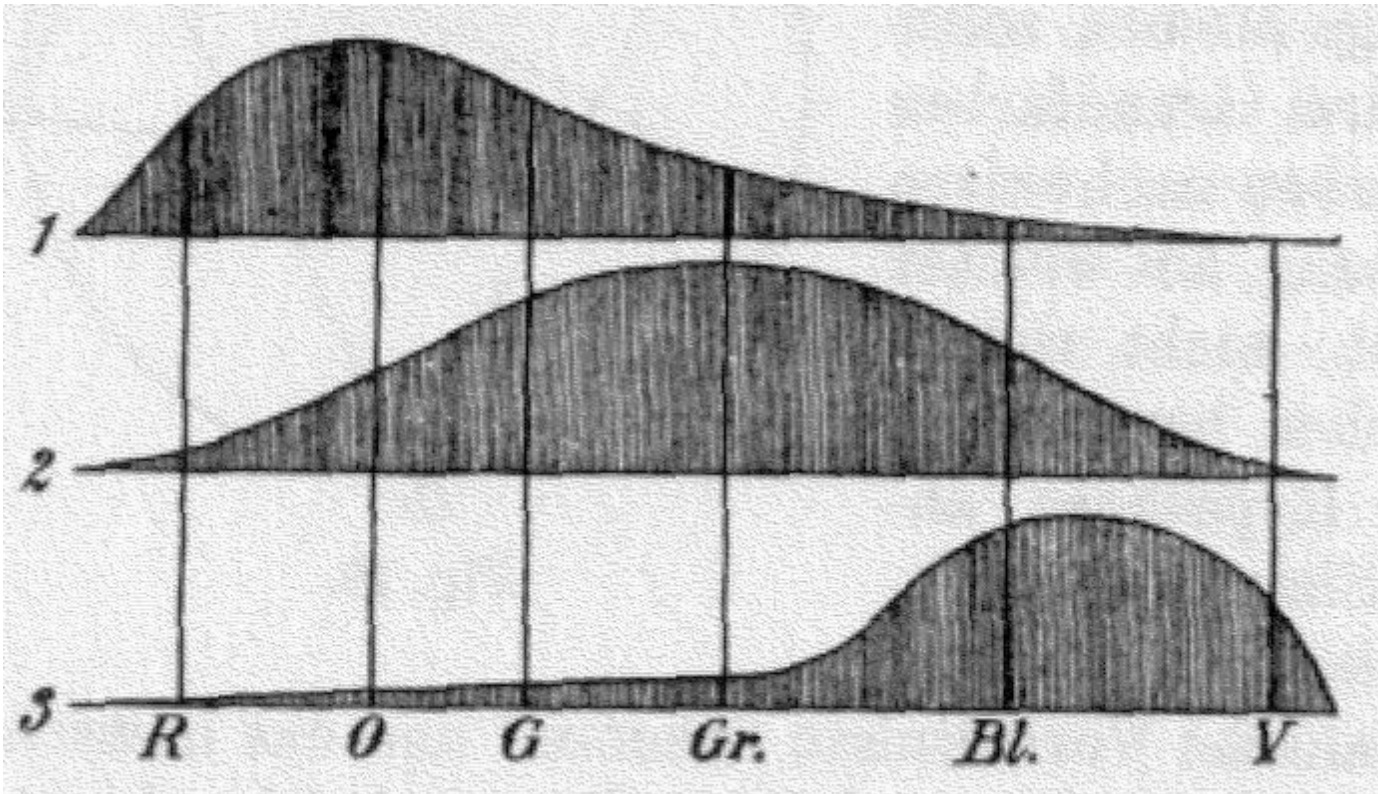


Figure 3.2.4 (reproduced). *Helmholtz's own drawing of the three sensitivities, from the* Handbuch der physiologischen Optik. *The horizontal axis is the spectrum, labeled in German with the color names of the day: R for* Roth, *O for orange, G for* Gelb *(yellow, not green), Gr for* Grün, *Bl for* Blau, *V for* Violett. *Curves 1, 2 and 3 are the three receptor types, drawn broad and heavily overlapping, each one responding across most of the spectrum with a different peak. Compare it with the measured curves in Figure 3.2.1: the shapes were guessed rather than measured, and curve 3 in particular is far too narrow, but the structure is right, and it was right roughly seventy years before anyone could record from a single cone. Everything in this chapter is a refinement of this picture. Image: Hermann von Helmholtz,* Gesammelte Schriften *III.2 (Olms-Weidmann, 2003), Fig. 21, by 1894; public domain, via Wikimedia Commons.*

For photopic color matching, a fixed observer is modeled by three cone excitations (r_L, r_M, r_S). Equal excitations predict a match when spatial, temporal, and adaptation conditions are held fixed. Rod contributions, receptor differences between observers, and contextual appearance require additional considerations.

Sidebar 5: who was von Helmholtz?

Hermann von Helmholtz (1821–1894) was the physicist and physiologist who turned Thomas Young's guess into a working theory. Young had argued decades earlier that the eye cannot possibly carry a separate detector for every wavelength and must instead make do with a few receptor types; Helmholtz developed this quantitatively, and the three-receptor account of color vision is still called the *Young–Helmholtz* theory. His *Handbook of Physiological Optics* founded the field, and he invented the ophthalmoscope that let physicians look into the living eye. Two of his ideas run straight through this chapter. The first is trichromacy itself, the L/M/S projection that everything here is built on. The second is *unconscious inference*, his claim that perception is not a direct readout

of the retina but an inference the brain makes about the most likely cause of its input. That is exactly the stance we take later on color constancy, white balance, and "the dress," where the visual system guesses the illuminant and quietly discounts it. Beyond vision he is everywhere in physics as well, from the conservation of energy to the Helmholtz free energy and the acoustic resonators that carry his name. *Portrait: 19th-century photograph, public domain (via Wikimedia Commons).*

To compute the integral, sample an energy spectrum at wavelengths λ_i with bin widths $\Delta\lambda_i$. Put the spectral-density values into $\mathbf{E}$, and define the matrix entries as $C_{ki} = c_k(\lambda_i)\Delta\lambda_i$. Then

$$\mathbf{r} \approx \mathbf{CE}, \qquad r_k \approx \sum_{i=1}^{N} E(\lambda_i)c_k(\lambda_i)\Delta\lambda_i. \tag{5}$$

Each term is the excitation contributed by one wavelength bin. The bin width matters: doubling the number of samples should improve the approximation, not double the response. With nonuniform samples, use the appropriate quadrature weights. Figure 3.2.5 illustrates the matrix computation.

```
response = [0, 0, 0]
for each wavelength bin i:
    for each cone type k:
        response[k] += spectrum[i] * sensitivity[k, i] * bin_width[i]
```

Keep spectrum and sensitivity on the same wavelength grid and use one wavelength unit throughout.

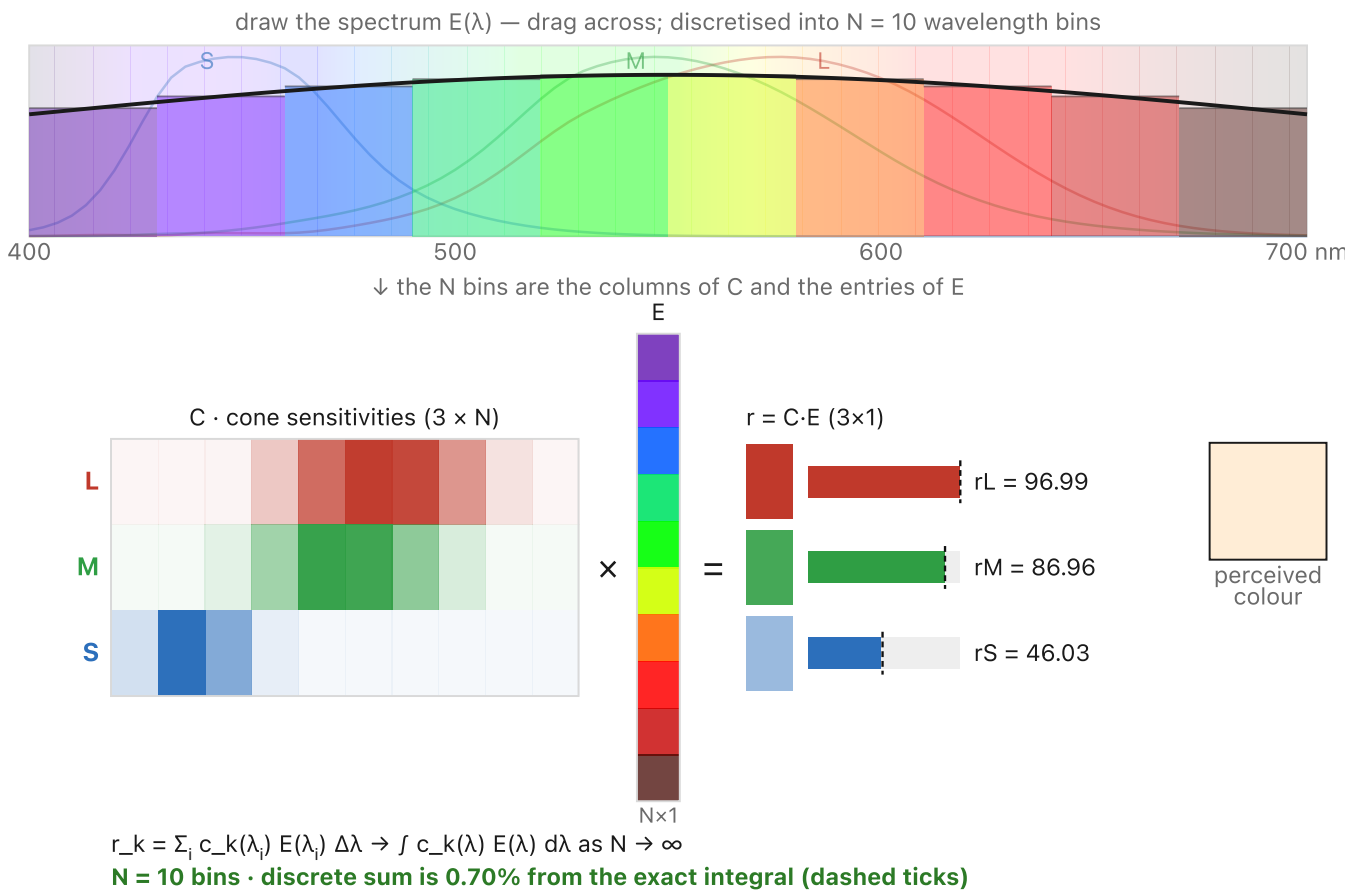


Figure 3.2.5 (reproduced). *Cone response as a matrix–vector product* $\mathbf{r} = \mathbf{C}\,\mathbf{E}$. *The spectrum is discretized into* N *samples (the vector* $\mathbf{E}$*); the three cone sensitivities are the rows of the* $3 \times N$ *matrix* $\mathbf{C}$*; the product is the three cone responses. In the limit* $N \to \infty$ *the spectrum is infinite-dimensional and the sum becomes an integral. Interactive: draw the spectrum by dragging across the plot, and move the bins slider, and as N grows the staircase sum (a Riemann sum) hugs the continuous curve.*

> 💡 **BIG LESSON (L3.1) — A CONE RESPONSE IS A DOT PRODUCT OF THE SPECTRUM WITH THE CONE'S SENSITIVITY**
>
> Each cone excitation is an integral of spectral density times the appropriate sensitivity. A sampled implementation must include wavelength-bin widths (or absorb them into the matrix). Three excitations summarize photopic color matching for a fixed observer and viewing condition; they do not recover the spectrum.

We established above that a single cone cannot distinguish wavelengths. Only by *comparing* the three cones' outputs can the brain begin to separate the two. However, this ability is limited. This is unsurprising because trichromatic vision summarizes a whole spectrum with only three numbers.

The three-bin examples below are a model of the spectrum, not three isolated cone channels. A bin containing long-wavelength light generally excites both L and M cones. Changing its power changes their ratio as well as their

absolute responses. Equal bin powers need not produce equal cone excitations or a neutral white: those depend on the bin boundaries, sensitivity normalization, and reference white. The displayed swatches show an RGB approximation of the resulting color, limited by the screen's gamut.

A few cases make the three-number code concrete. Put the power in the long-wavelength bin and L responds most, M somewhat less, S hardly at all: red. Put it in the short bin and S leads: blue. In the middle bin, M leads with L close behind: green. Fill the long and middle bins together, with little short, and L and M both respond strongly, with L further ahead than for the middle bin alone: yellow, a color with no bin of its own, which is the sign that yellow sits next to green in cone space. All three bins at comparable power give a color near white, depending on the normalization. Orange (mostly long, some middle), cyan (middle plus short) and purple (long plus short, skipping the middle) fill in the hue circle. Every named color is a ratio of three cone catches, and very different spectra can land on the same ratio.

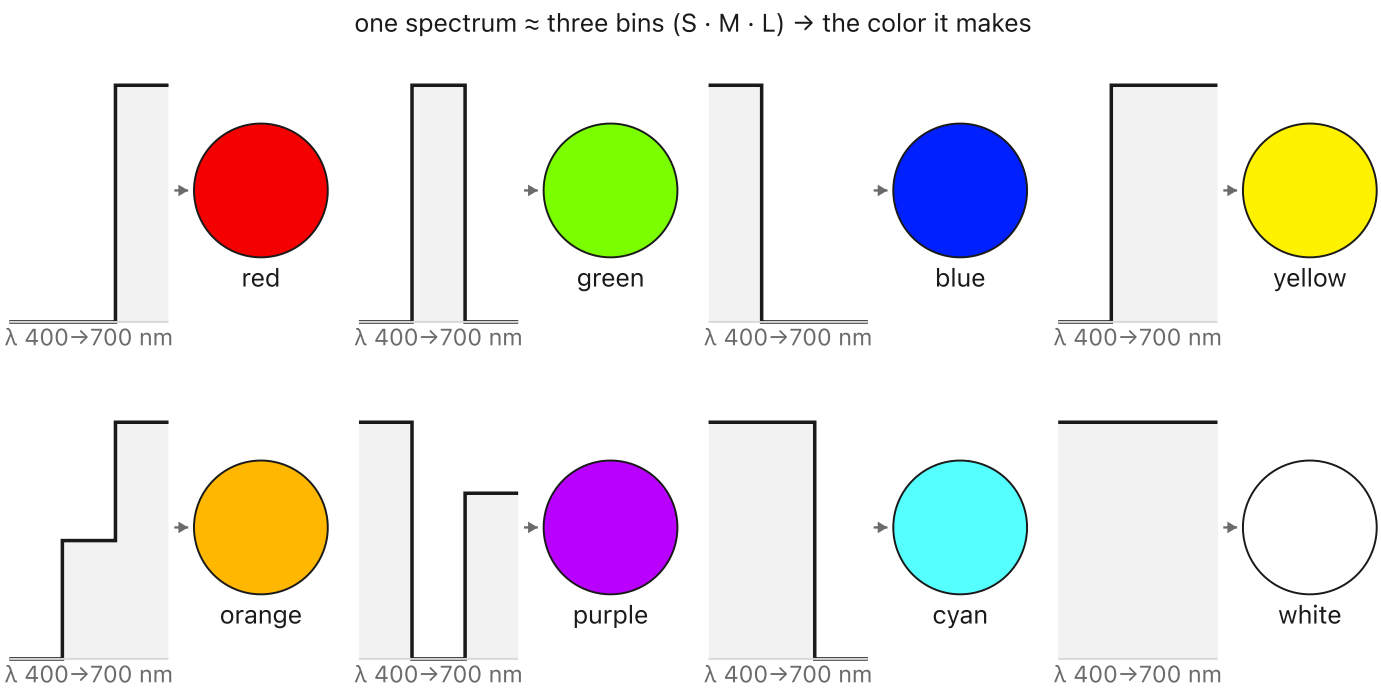


Figure 3.2.6 (reproduced). *The crudest color model that works. Chop the visible band into just three bins (S, short, roughly blue; M, medium, roughly green; L, long, roughly red) and set a flat power in each. Even this three-number cartoon of a spectrum already makes a recognizable color: yellow is medium plus long, cyan is short plus medium, purple is short plus long, and white is all three at once. Each color disk is computed from its three bins by integrating the CIE color-matching functions over them, not hand-picked, so it shows the same additive logic the three cone types run, reduced to three numbers. Mix the bins yourself in the companion Figure 3.2.7.*

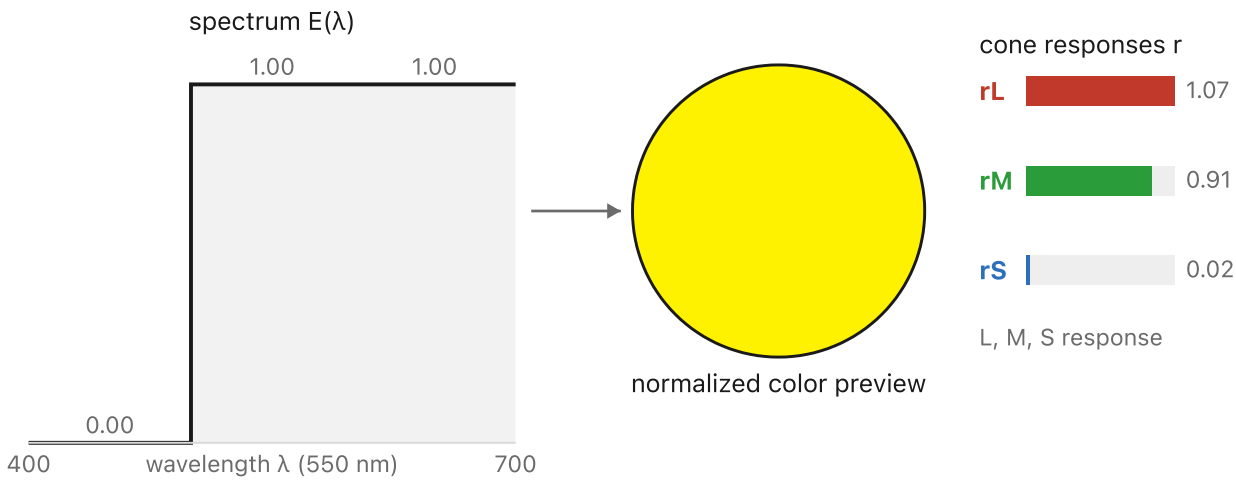


Figure 3.2.7 (reproduced). *Mix a color from three numbers. Drag the three bin powers (S short and blue, M medium and green, L long and red) and the perceived color updates, along with the three cone responses r = (r_L, r_M, r_S) those bins drive. Load a preset to see that yellow is just M plus L, cyan is S plus M, purple is S plus L, and white is all three. Trichromacy in miniature: any spectrum you can make here is summarized by exactly three numbers, and any two that give the same three cone responses look identical.*

Metamers

in the book ↗

Different nonnegative spectra can produce the same cone-response triple. Such spectra are *metamers* for that observer and viewing condition. In the sampled model, a spectral perturbation in the null space of **C** leaves the responses unchanged, provided the modified spectrum remains nonnegative (Figure 3.2.8). Displays exploit this: they reproduce an in-gamut color by emitting a metamer rather than the original spectrum. Three fixed physical primaries cannot reproduce every visible color (Chapter 4.1, Linear color reproduction and CIE XYZ).

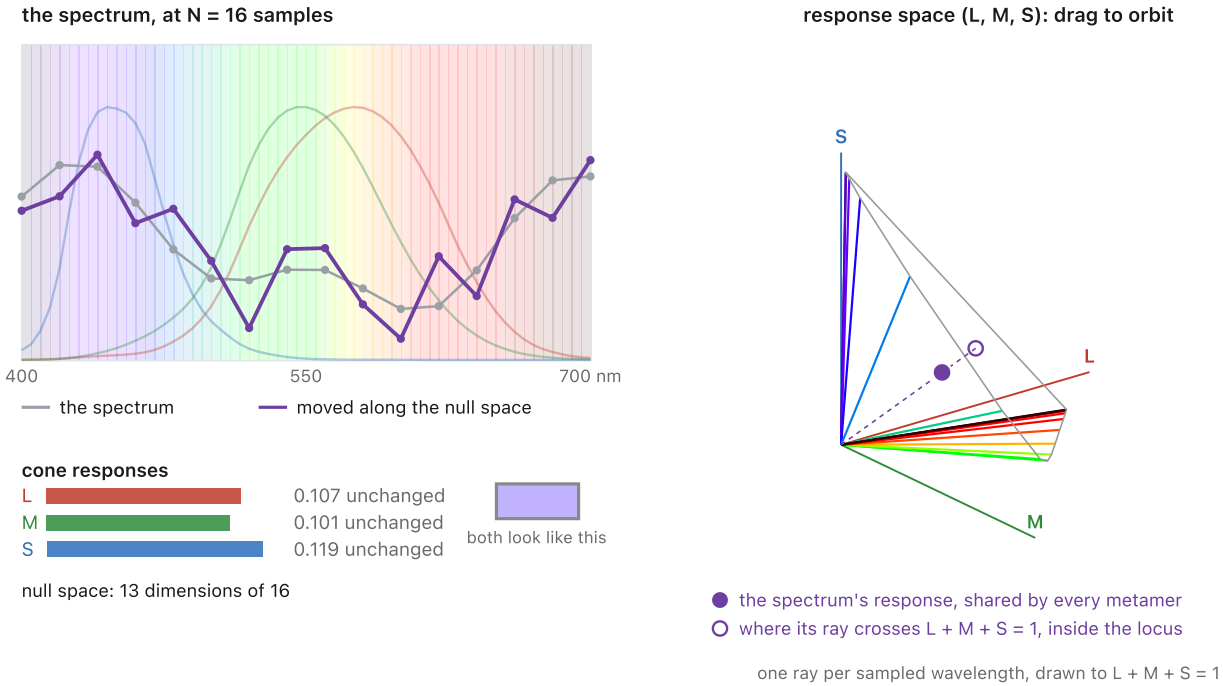


Figure 3.2.8 (reproduced). *The eye as a projection (interactive). A spectrum sampled at N wavelengths is a point in N dimensions, and the three cones map it to three numbers. Everything the cones send to zero, the null space of* $\mathbf{C}$*, has $N - 3$ dimensions: move the spectrum along one of those directions with the slider and the curve changes while the three responses and the color do not. The slider stops where some sample would go negative, since past that point the curve is no longer a light. On the right is response space: each ray is one sampled wavelength, drawn to where it crosses $L + M + S = 1$, so the tips trace the spectral locus, and every spectrum's ray crosses inside it. Drop a cone and response space flattens to a plane; a second spectrum that differed only in that cone's response lands on the same point, which is why a dichromat confuses colors a trichromat tells apart.*

💡 BIG LESSON (L3.2) — METAMERS: DIFFERENT SPECTRA, SAME CONE RESPONSE

Different spectra can yield the same cone excitations for a specified observer. This enables color reproduction without reproducing the spectrum, but a fixed display can match only colors within its gamut. Matches between reflecting materials can fail when the illuminant changes.

Sidebar 6: a color outside the gamut: stimulating one cone type directly ("olo")

Overlapping cone sensitivities limit the response combinations obtainable by changing a spatially uniform spectrum. Fong, Doyle, Wang and colleagues used adaptive optics, cone classification, and eye tracking to direct light preferentially to selected cone types Fong et al. 2025 (Oz / "olo") Roorda & Williams 1999. Observers reported a highly saturated blue-green, named “olo,” outside the gamut of conventional spatially uniform spectral stimulation. The system changes where light lands on the receptor mosaic; it still uses light. A conventional display cannot reproduce that experience. See Chapter 37.6, Retinal projection.

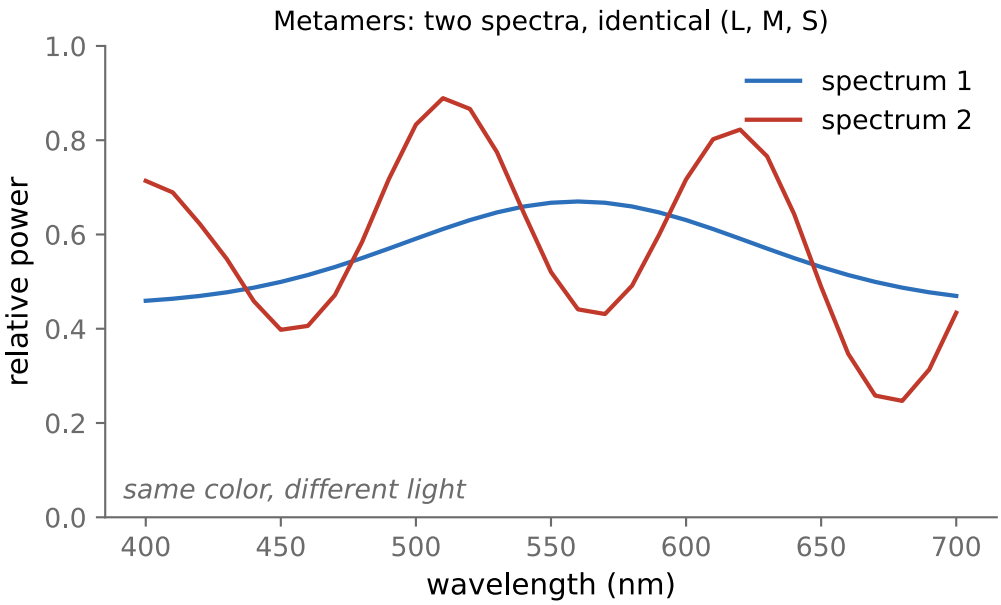


Figure 3.2.9 (reproduced). *Two metamers. The two spectra (left) are completely different functions of wavelength, yet they project to the same three cone responses and so look like the same color (right). Metamerism (a direct consequence of the three-number projection) is what lets a three-primary display match almost any color.*

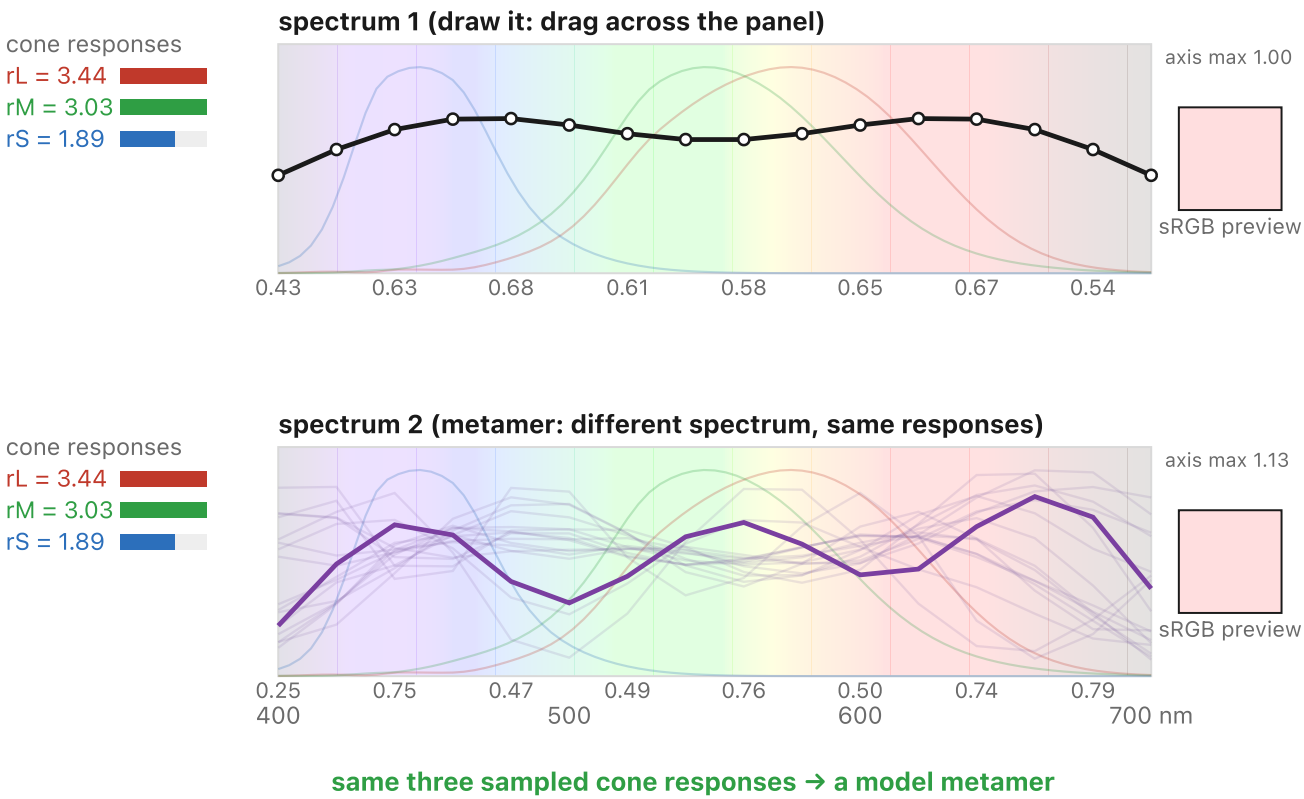


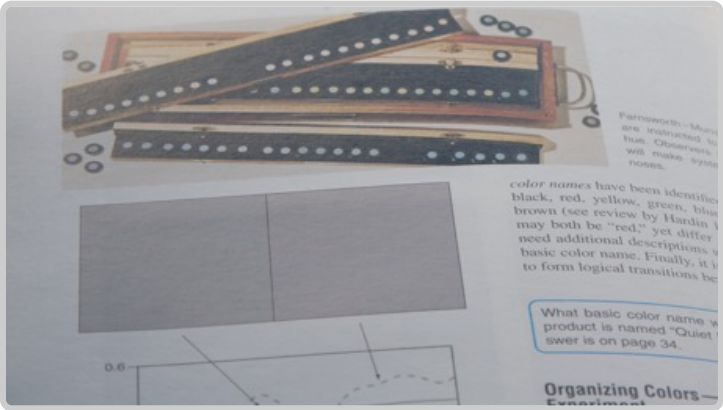
Figure 3.2.10 (reproduced). *Metamerism, made constructive. The top spectrum is projected onto the three cone curves to give the responses* (r_L, r_M, r_S)*; the bottom spectrum is built to be different at every wavelength yet share the same three responses, so both collapse to the same triple and the same perceived color. Generate a fresh metamer, switch families (smooth, spiky with a chosen number of spikes, or block/step), or overlay many family members, all sharing identical cone responses. Many spectra, one response.* Interactive: draw the top spectrum by dragging across the panel; the bottom spectrum is generated to be different at every wavelength yet share the same three cone responses.

Sidebar 7: metamers at the movies: Dolby 3D

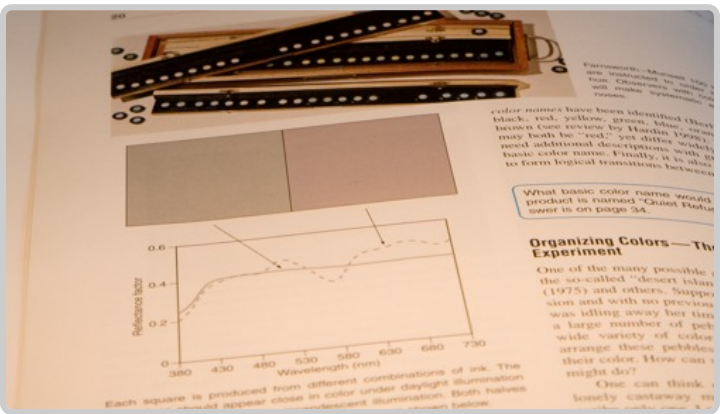
Dolby 3D uses separate spectral bands to deliver different images to the two eyes. Each eye receives its own red, green, and blue primary bands through matching filters Jorke & Fritz 2003 (Infitec / Dolby 3D). Color correction seeks to match the two eyes' rendered colors within the gamuts the primary sets share. The individual red bands need not be metamers by themselves: the match uses mixtures of each set of primaries. Filter leakage, viewing angle, and observer differences limit the separation and color match.

Two fabrics can match in a shop and differ outdoors. For a nonfluorescent surface in the simple reflectance model, the spectrum reaching the eye is proportional to illuminant times reflectance. A change of illuminant weights the reflectance differences differently, so equality of the three integrals need not survive (Figure 3.2.13). Narrow spectral peaks can make matches sensitive to particular reflectance differences, but smooth illuminants can also break a match. Color-critical work specifies the illuminant and observer. Figure 3.2.11 shows a printed metameric pair under daylight and halogen illumination.

Illuminant metamerism, photographed: two patches that match under daylight no longer match under a halogen lamp. **Click a photo to enlarge.**



under daylight, the two gray patches match



under halogen the match breaks (right patch goes pink)

Photos credited in the book's image credits.

Figure 3.2.11 (reproduced). *Illuminant metamerism, photographed. Two gray patches printed as a metameric pair in Billmeyer & Saltzman's* Principles of Color Technology *(their differing reflectance curves are shown on the page). Left: under daylight the two patches read as the same neutral gray. Right: under a warm halogen lamp the match breaks: the right patch takes on a pink cast while the left stays gray. A metameric match is a match between two products (reflectance × illuminant); change the light and the products pull apart, and the camera records the same divergence the eye sees.*

No community examples yet — be the first to contribute.

+ Contribute your example

Figure 3. COMMUNITY Upload your own photo of **metamers** (colors that match under one light but split apart under another) to share with other readers.

💡 **BIG LESSON (L3.3) — METAMERISM CAN BREAK WHEN THE ILLUMINANT CHANGES**

A metameric match is a match between two **products** of reflectance and illuminant (**L2.6** show). Swap the illuminant and you multiply by a different spectral shape, so two surfaces that looked identical under one light can **diverge under another**: *illuminant metamerism*. It is the headache behind prints that shift between the proofing booth and the living room, and fabrics that match in the shop but clash in daylight; color-critical work must always name the light a match is judged under.

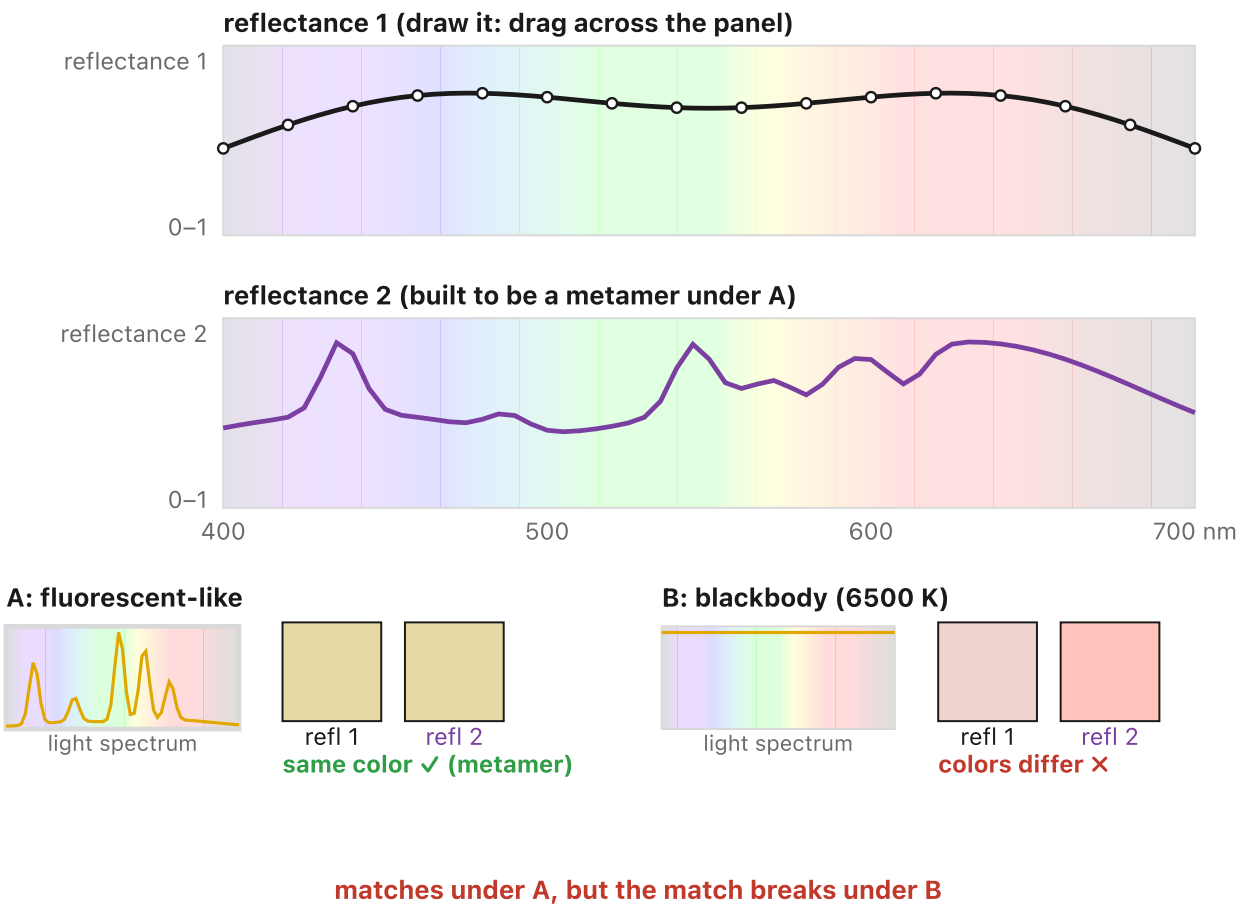


Figure 3.2.13 (reproduced). *Metamerism depends on the light (interactive). Two **reflectance** spectra are built to be metamers under one illuminant, say fluorescent: multiplied by that light they land on the same three cone responses, so they look identical. Under a second illuminant, say daylight, the same two reflectances multiply by a different spectral shape and their colors pull apart. Draw your own reflectance and press "new metamer" for a fresh partner that matches under illuminant A; the two swatch pairs show the match holding under A and breaking under B. Press "new random metamer" for an ordinary partner, or "new adversarial metamer" for a strong mismatch, a partner chosen among the metamers of A by a projected power iteration to change the response under B as much as it can find; "new adversarial reflectance 1" goes further and moves reflectance 1 itself toward the edges of the physical [0,1] range, so its partner can diverge further. Both search for a large mismatch; neither is a proved maximum under the [0,1] constraint. The "match under both" checkbox does the opposite, much harder job: find a reflectance that matches under **both** lights at once, a perturbation that lives in the null space of both illuminants. That extra constraint is exactly the challenge of printing, where an ink set mixed to match a target under the proofing light drifts under the customer's bulbs, and matching everywhere costs many more degrees of freedom.*

Consider a printer matching a fabric. The inks and the fabric can have quite different reflectance spectra while sending the same three cone responses to an observer under the proofing light. Moving both under another light changes the wavelength-by-wavelength weights before those responses are summed. The differences that canceled under the first light need not cancel under the second. A narrow spectral peak can emphasize a region where the two reflectances differ; a smooth change of illumination can also break the match. This is why specifying only three color coordinates does not promise a match under every illuminant. A proofing agreement needs a viewing illuminant and observer as well as a numerical tolerance.

Linear color reproduction and CIE XYZ

in the book ↗

A color match reproduces an observer's responses to light without reproducing its spectrum. For a fixed standard observer, tristimulus coordinates describe these matches with three numbers. The central calculation is linear, but its physical constraints matter: a display can add light from its primaries, not subtract it.

The color reproduction perspective

OPTIONAL
in the book ↗

In principle, measuring color and reproducing it are separate problems. Given light, measurement reports coordinates for its color. Given those coordinates, reproduction builds light on a screen or a page that matches it. You could imagine a camera reporting device-independent measurements to a separate display module, with neither module needing to know how the other works. Much of the notation we actually use comes from treating them together as a reproduction pipeline.

The reason is historical and experimental. Color matching provided a way to measure perceptual color before cone sensitivities could be measured directly. An observer adjusted three fixed primary lights until their mixture matched a test light. The three knob settings were the measurement. These coordinates therefore described amounts of particular reproduction primaries, rather than direct cone responses. Changing the primary lights changed the coordinates of the very same test light. The experiment made measurement possible, but also made the reproduction apparatus part of the measurement convention. Television reinforced the practical importance of a primary-based description: a central engineering task was to get the right picture out of a particular screen.

Metamerism makes this useful. Different spectra can match for the specified observer and conditions. They may cease to match for another observer, or after a change of illumination when the spectra come from reflecting objects. Colorimetry describes the match; it does not by itself predict appearance across different surrounds and adaptation states.

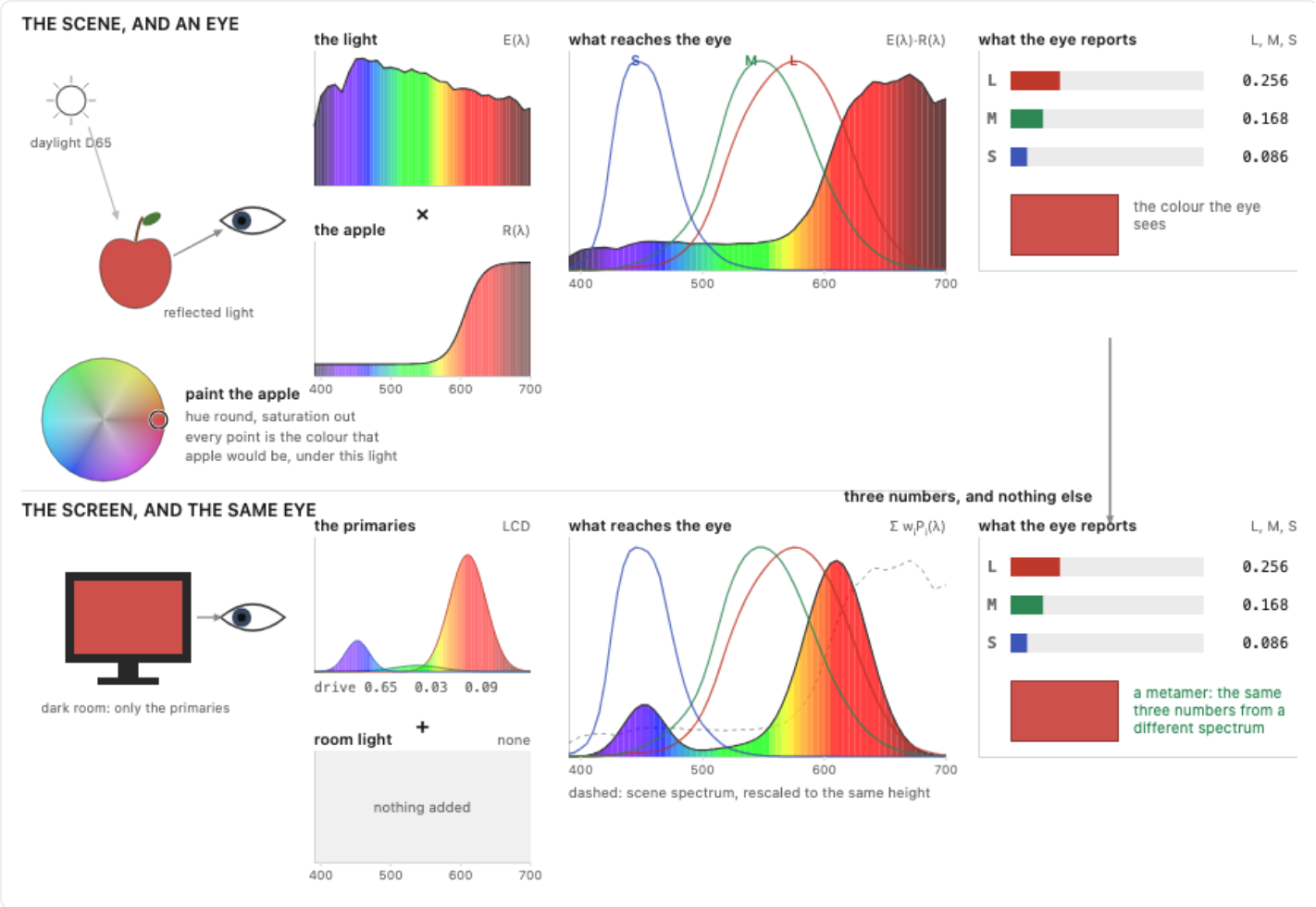


Figure 4.1.1 (reproduced). *Color reproduction end to end, twice over (interactive). **Top**, a lamp's spectrum is* multiplied *by the apple's reflectance, and what arrives at the eye is projected onto three cone sensitivities. Three numbers, and every other fact about that spectrum is gone. **Bottom**, three display primaries, driven by three weights,* add *to a spectrum that looks nothing like the first one and lands on the same three cone responses. The swatches match; the spectra plainly do not. That is metamerism, and it is the only reason a screen with three primaries can stand in for a world of spectra it cannot emit. Nothing here is drawn by hand: the illuminants are the measured CIE tables at 5 nm, the cone curves come from the 1931 observer, and the weights are found by solving the* 3×3 *system that puts the display's tristimulus on the scene's, so the figure is free to fail and the failures are the lesson. Ask for a color outside the primaries' triangle and a weight goes **negative**: the screen clips and the match visibly breaks, which is* gamut. *Turn on a room light and its reflection off the screen **adds** to what the primaries emit, breaking the match again, which is why color-critical work happens in a dim room. Swap the primaries from an LCD's broad filters to a laser projector's narrow lines and the reachable set grows while the spectrum sent to the eye becomes even less like the scene's: wider gamut, wilder metamer. **White balance** is the one extra step a camera takes. With it off, the screen reproduces the cone triple the eye actually received, so an apple under tungsten arrives orange; with it on, the triple is adapted to each side's white and the apple looks like an apple. Both are correct answers to different questions, and knowing which one you are asking is most of color management. Exposure is set so the picture fits the screen, so only chromaticity is ever left to fail. Paint the apple on the wheel under the scene: hue is the angle, saturation the radius. It is not a generic color wheel, but a map of what that apple would actually be under the light currently selected, every point computed through the same reflectance-times-illuminant-times-cone-curves path as the rest of the figure, so the wheel itself warms under tungsten and goes strange under the fluorescent*

tube. The colors on it are also gentler than a display's primaries, which is the truth about surfaces: a real reflectance is broad and smooth, and broad reflectances cannot be very saturated.

XYZ provides a common reference for device characterization and color conversion. The agreement matters as much as the matrix algebra. With a shared reference, a new display can describe its primaries once instead of requiring a separately standardized conversion from every existing camera or working space. Conversion between two linear tristimulus coordinate systems for the same observer is a matrix multiplication. The shared reference lets those conversions be composed. A camera's three raw channels generally do not span exactly the standard observer's matching functions, so its color-correction matrix is an approximation fitted to a calibration set. Nonlinear encodings must be decoded before applying linear-light color matrices.

Changing illumination is another limitation. The illumination multiplies reflectance wavelength by wavelength, whereas a three-channel measurement has already discarded most of that spectral information. A three-by-three chromatic-adaptation transform is a useful model of corresponding colors, not a general solution to spectral relighting. Smooth illumination alone does not make it exact.

A typical rendered-image path includes black-level correction, white-balance gains, demosaicking, camera color correction, tone and color rendering, and output encoding. Implementations combine or reorder some stages. XYZ often enters through the calibration that defines the matrices rather than as a separately stored image. To convert between two linear RGB systems, we can compose an RGB-to-XYZ matrix with an XYZ-to-RGB matrix and apply the resulting matrix directly. The intermediate XYZ coordinates define the conversion without needing to be written into an image buffer. This is why XYZ can be central to a system whose working images are all called RGB.

Analysis vs synthesis and non-orthogonality

in the book ↗

First imagine three nonoverlapping, flat sensitivity bands. Measuring the power in each band and emitting the same power within that band would reproduce the three measurements. Human cone sensitivities overlap instead: a light chosen to stimulate one cone type also stimulates others. If we use the sensitivity curves themselves as emission spectra, then scale them by the measured responses, measuring the result applies that overlap a second time. The responses generally change, as Figure 4.1.3 shows. We need to account for how every chosen primary stimulates every analysis channel.

The failing construction is short enough to write out. Emit the sensitivity curves weighted by the responses, $\hat{\mathbf{e}} = C^\top \mathbf{r}$, and measure the result: the responses come back as $C\hat{\mathbf{e}} = CC^\top \mathbf{r}$. The three-by-three matrix $G = CC^\top$ holds the overlap of every pair of curves, and it is the identity only if the curves are orthonormal. For cones it is far from that, above all for L and M, which overlap heavily, so a light that should excite mostly L comes back measured as a large L and M mixture, and a monochromatic test light is rebuilt as a broad, desaturated one (Figure 4.1.3). The correct weights are $G^{-1}\mathbf{r}$, which is what the dual basis computes, and they include negative entries: to reproduce the test light with these curves as emitters we would have to subtract light, which is the same wall that real primaries hit in the next section.

Sample an incoming spectrum into a vector $\mathbf{e}$. Let C be a three-by-N analysis matrix, including wavelength-bin weights, and let the columns of P be three sampled display-primary spectra. Measurement and reproduction are

$$\mathbf{r} = C\mathbf{e}, \qquad \hat{\mathbf{e}} = P\mathbf{p}, \qquad CP\mathbf{p} = \mathbf{r}. \tag{8}$$

Thus matching the target requires solving $A\mathbf{p} = \mathbf{r}$, where $A = CP$. Each column of A gives the three measured responses to one display primary. If A is invertible, there is a unique algebraic solution. The display can realize it only if every primary weight lies within its allowed range.

💡 **BIG LESSON (L4.2) — COLOR ANALYSIS AND SYNTHESIS REQUIRE DIFFERENT SPECTRAL DISTRIBUTIONS**

Sensitivity curves measure a spectrum; emission primaries construct one. Reusing the sensitivity curves as primaries and the measured responses as weights generally gives the wrong color. Calibrate how each primary stimulates the observer, then solve for its weight.

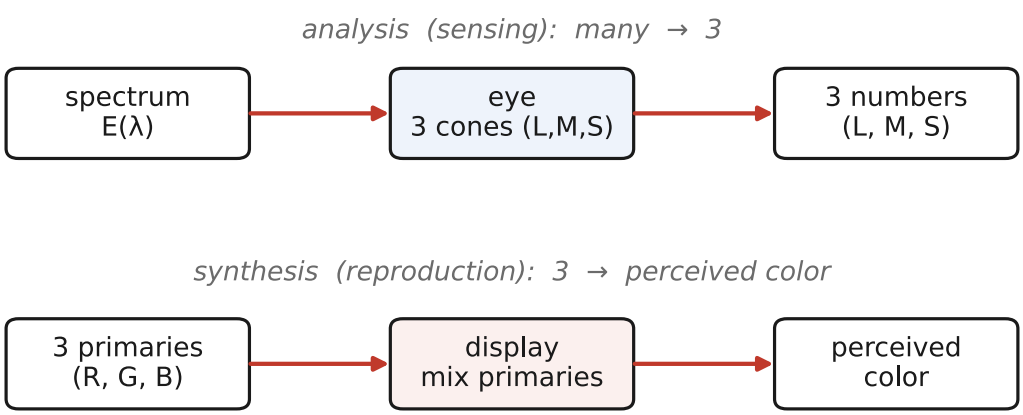


Figure 4.1.2 (reproduced). *Analysis versus synthesis. On the left, analysis: a full spectrum enters and is projected onto three sensor responses (sense → 3 numbers), the arrow pointing inward. On the right, synthesis: three numbers drive a small set of primaries that add up to a reproduced spectrum (reproduce ← few primaries), the arrow pointing outward. The two operations are mirror images, and most color devices are one or the other.*

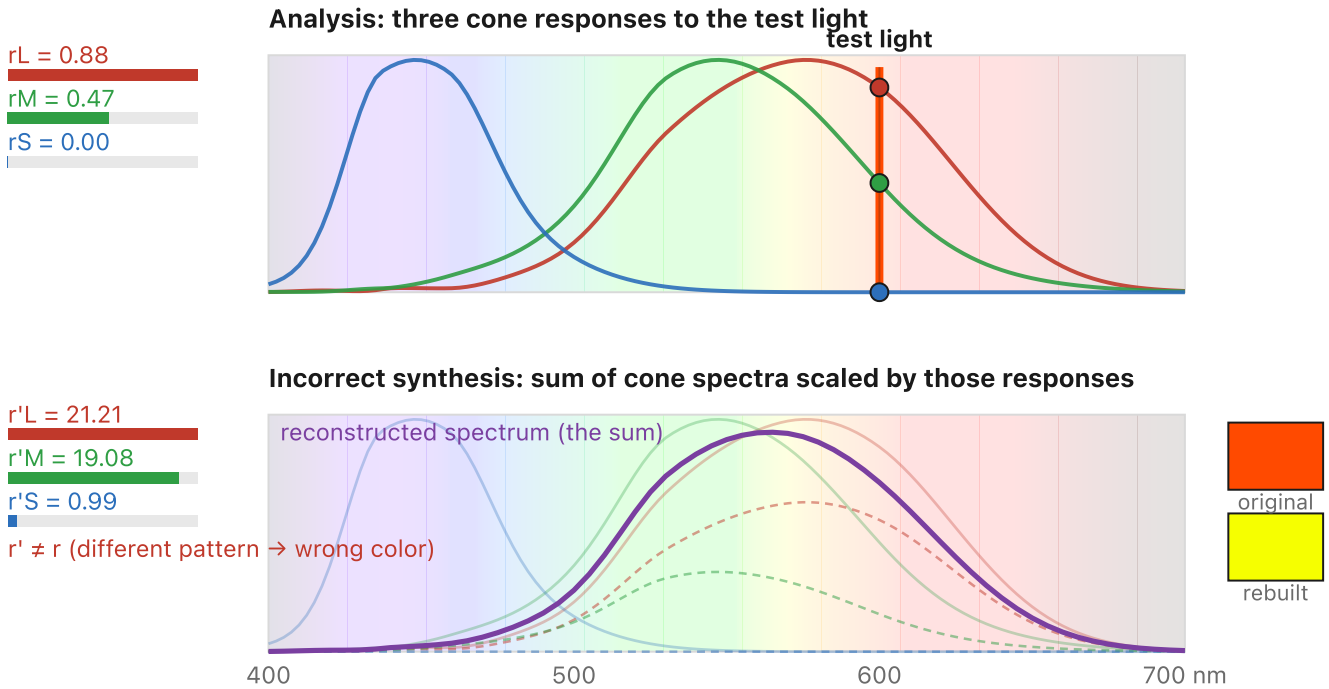


Figure 4.1.3 (reproduced). *Why the cone curves are the wrong synthesis primaries (interactive). Pick a monochromatic test light with the slider. Top (analysis): the three cone responses are the heights of the L, M, S curves at that wavelength. Bottom (incorrect synthesis): rebuild the light by scaling each cone spectrum by its own response and summing them. The sum is a broad, washed-out spectrum, and when it is measured back through the cones its responses are not the originals (they are the original responses multiplied by the cone overlap matrix), so its perceived color is wrong. Reusing the analysis basis for synthesis fails, which is why real display primaries are not the cone sensitivities.*

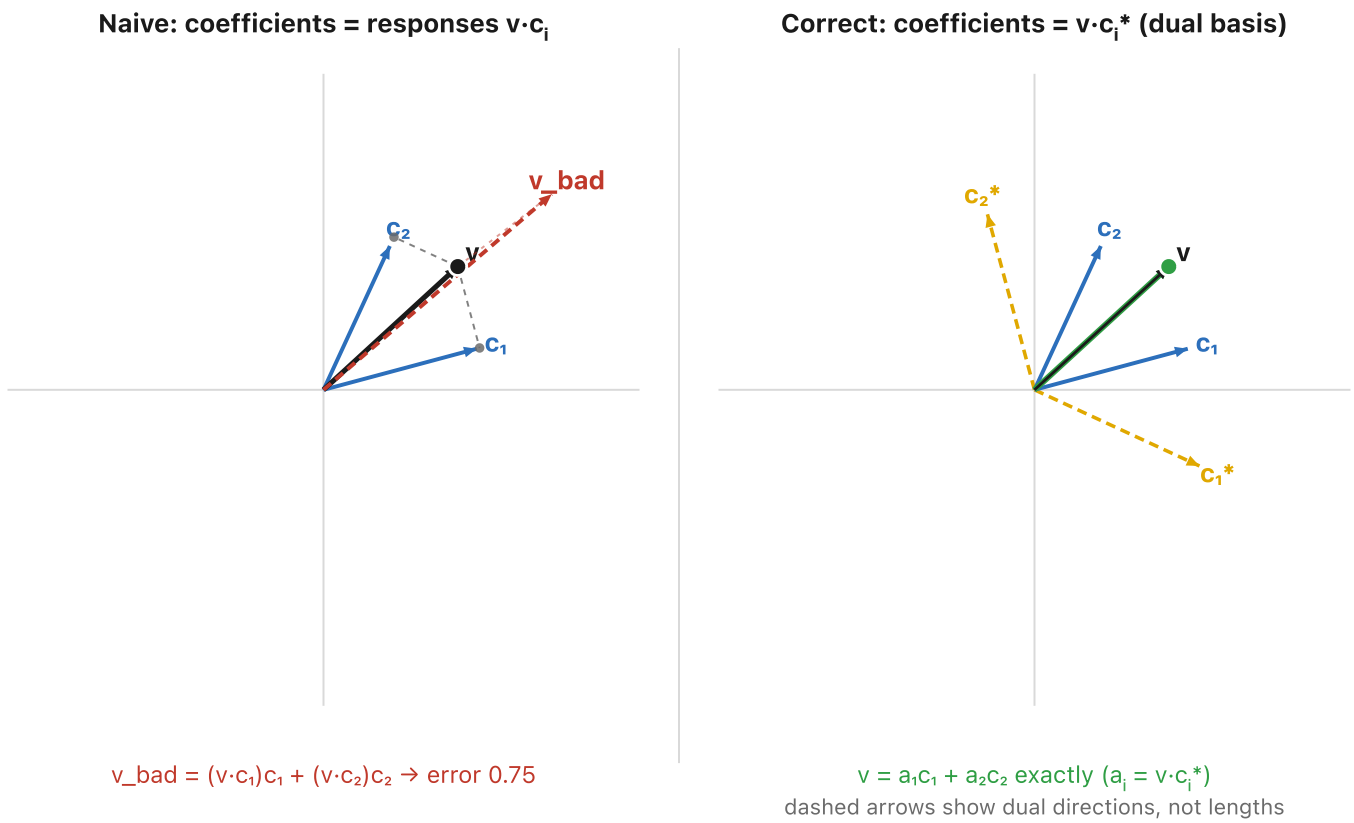


Figure 4.1.4 (reproduced). *Reconstruction in a non-orthogonal basis, wrong and right (interactive). Drag the basis vectors c_1, c_2 or the target v. Left: the naive attempt uses the analysis responses $v \cdot c_i$ directly as coefficients, which only works for an orthonormal basis, so the rebuilt vector misses the target. Right: the correct coefficients come from the dual (reciprocal) basis c_1^*, c_2^* (defined by $c_i^* \cdot c_j = \delta_{ij}$); with them v is reconstructed exactly. The dual vectors lean the opposite way, into the negative directions, so recovering the coordinates needs quantities the analysis basis cannot supply, which is exactly why a sensor's cone-like responses cannot double as a display's primaries. Only for an orthonormal basis does the dual coincide with the basis and analysis equal synthesis.*

The problem color reproduction actually solves is not quite the textbook one. In linear algebra, reconstruction means rebuilding a vector *exactly* as a linear combination of basis vectors, so that the result equals the original. Color reproduction asks for less. We do not need the reproduced spectrum to equal the original spectrum, only to produce the *same three cone responses*, that is, to be a metamer of it. That is a weaker requirement, a match of projections rather than of whole vectors, and it buys a great deal of freedom: the reproduction primaries can be chosen independently of the analysis (cone) vectors, they can be picked to be physically realizable lights, and an in-gamut color can still be matched. Solving the primary-response system gives the coefficients for a given set of primaries; metamerism is what lets us choose those primaries in the first place.

This is where the usual non-orthogonal-basis intuition helps, provided we apply it to the right space. The three primary-response columns need not point along independent cone axes. Solving their linear system compensates for the cross-talk. Merely using a measured cone response as the weight of a similarly shaped emitter applies the cross-talk again. The distinction between taking dot products and solving for synthesis coefficients is familiar from dual bases in linear algebra; here the goal is equality of three measurements, with no requirement to reconstruct the original spectrum.

The algebraic solution can include a negative primary weight. That is a useful coordinate, but a display cannot emit negative light to subtract the unwanted response of another primary. Its physically available mixtures form a restricted set. Metamerism gives us freedom to use a different spectrum for a match; it does not ensure that a particular display's positive mixtures contain a match for every visible color.

The representation labels are a practical type system. A matrix accepting linear RGB cannot be applied directly to encoded RGB merely because both have three components. A luminance and a luma can both be called brightness in conversation while requiring different computations. Before composing two matrices, check that the first one's output has the second one's expected primaries, reference white, normalization, and encoding. The matrix dimensions alone cannot catch these mistakes. This is why color code needs more bookkeeping than an arbitrary three-vector calculation.

Label the representation at each transition. In this part, bare RGB denotes linear values, primes denote a specified nonlinear encoding, and a superscript ℓ denotes a specified log encoding. The curve must still be named: a prime does not distinguish sRGB, Rec. 709, or PQ. PQ can encode RGB as well as coordinates used inside ICtCp. A matrix signature states its input and output, for example $M : \mathrm{RGB}_{709} \to \mathrm{XYZ}$. Equal input and output types do not imply an identity operation; exposure scaling is an obvious counterexample.

Color matching and the most standard color space: CIE XYZ

in the book ↗

Why measure color through matching experiments rather than directly through cone responses? When the CIE established its 1931 standard, direct measurements of human cone spectral sensitivities were not available. Observers could nevertheless tell when two lights matched. That gave colorimetry an experimental route: define the analysis of a light by measuring its synthesis from adjustable primaries. Metamerism makes this possible because the spectra need not be identical, and the additivity of matches lets the measured coordinates be used for mixtures.

The design of that experiment is why it worked. Psychophysics, putting a number on a private sensation, is slippery: observers disagree, adapt, tire, and shift their answers with the surround and the wording of the task. A matching experiment asks for none of that. It is a *null* measurement: the observer reports only when the two halves of a field look the same, which people do far more reliably than they rate how different two colors are. Metamerism keeps the answer to three numbers rather than a spectrum, and linearity lets a limited set of measured matches extend to every mixture. The 1931 standard rests on the matching data of Wright and of Guild, about seventeen observers across the two studies, and it still predicts matches for typical observers well enough to anchor color measurement nearly a century later, within the limits of an average observer that the rest of this chapter keeps in view.

In a color-matching experiment, an observer views a test light beside a mixture of three fixed primaries. The primary amounts required for a match define its tristimulus coordinates. Some stimuli cannot be matched with nonnegative amounts: adding a primary to the test side is recorded as a negative amount on the mixture side. Repeating the experiment across wavelength gives the matching functions for those primaries. Wright and Guild used different primaries, and the CIE converted both data sets to three common monochromatic primaries at 700 nm, 546.1 nm and 435.8 nm Fairman, Brill & Hemmendinger 1997. The last two are emission lines of mercury, which any laboratory could reproduce exactly. The red was placed at 700 nm because in that part of the spectrum the hue hardly changes with wavelength, so a small error in setting it hardly changes the match. Spreading the three primaries far apart also keeps them well separated in cone response, the red and green especially, so the matching system is well conditioned.

0:00 / 0:03

Audio. Commission Internationale de l'Éclairage, the French name abbreviated CIE.

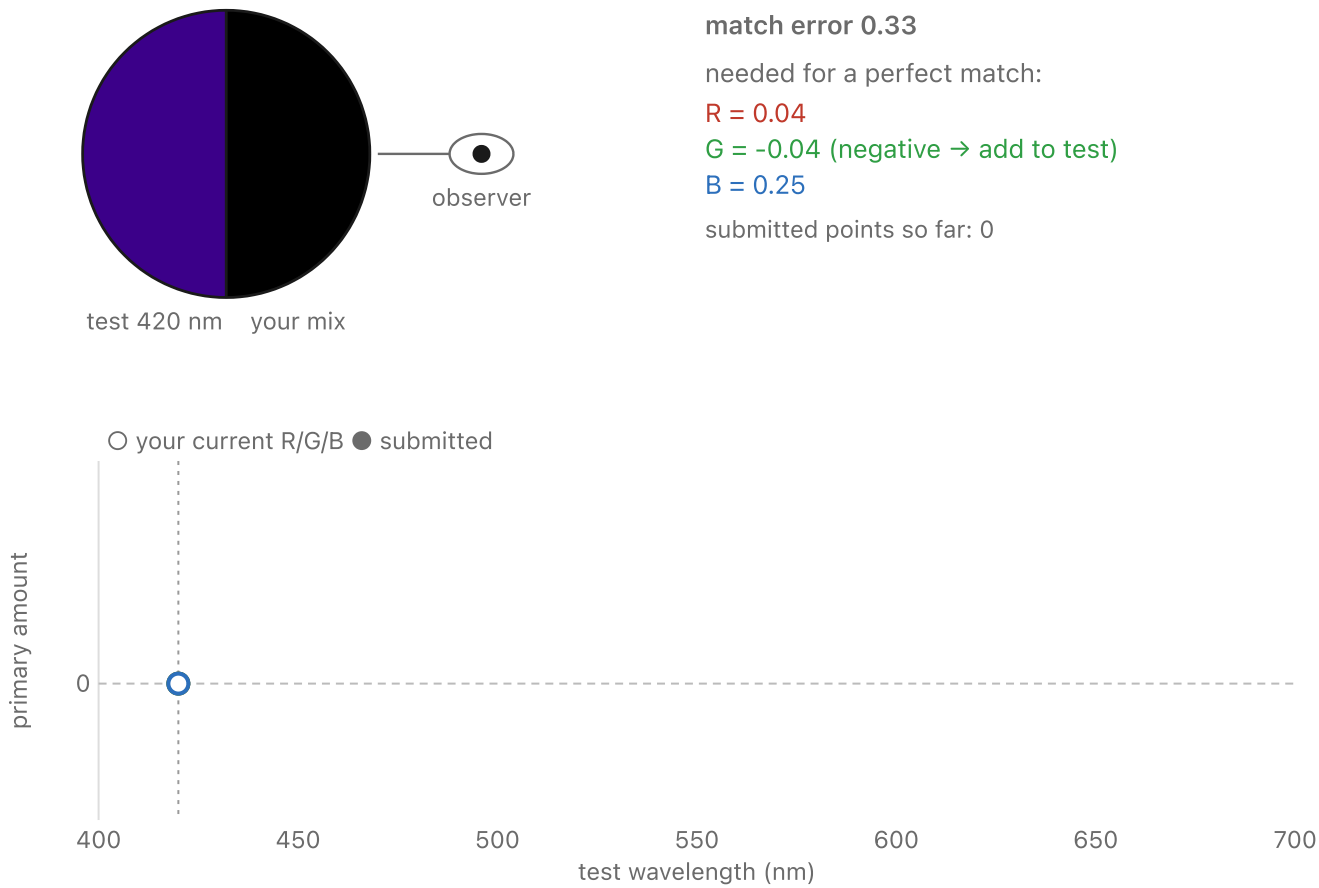


Figure 4.1.5 (reproduced). *The color-matching experiment, interactive. A bipartite disk shows a monochromatic test light (left) at a wavelength you choose against an additive mixture of three R, G, B primaries (right); turn the three sliders until the halves match, then submit to pin those three amounts onto the graph below; submit also advances the test wavelength by 50 nm and resets the sliders for the next match. Sweep the wavelength and repeat, and the pinned points, joined into a piecewise-linear curve, trace out the color-matching functions, including the blue-green stretch where the red amount must go negative (you add red to the* test *side), the impossibility that makes spectral colors unreachable by real primaries. The match works at all because of metamerism: the mixture need only fool the three cones, not equal the test spectrum. (Tick "show computed curves" to overlay the CIE 1931 matching functions, transformed to this figure's sRGB primaries rather than the historical CIE RGB ones.) The disk is drawn on your screen, which clips spectral colors it cannot show, so a visual match here is not a physical color-matching measurement; the numerical error compares the underlying tristimulus values.*

The CIE 1931 standard transforms the measured matching functions to imaginary XYZ primaries. These are coordinate axes, not three lamps. The transformation makes all three XYZ matching functions nonnegative and makes $\bar{y}(\lambda)$ equal to the standard photopic luminous-efficiency function. For a spectral stimulus $E(\lambda)$,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = k \int E(\lambda) \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} d\lambda. \tag{9}$$

The normalization k determines the scale. For relative tristimulus values, choose it so the reference white has $Y = 1$ (or explicitly 100). For spectral radiance in physical units, the photometric conversion gives Y in cd/m², with a factor approximately 683 lm/W and consistent wavelength units. Relative $Y = 1$ is not one cd/m². Neither version of luminance is a full prediction of perceived brightness.

For sampled spectra, include bin widths:

```
XYZ = [0, 0, 0]
for each wavelength bin i:
    XYZ += k * E[i] * [xbar[i], ybar[i], zbar[i]] * delta_lambda[i]
```

Use tables for the intended observer and field size. The CIE 1931 two-degree tables specify one standard observer, not every individual's cone sensitivities.

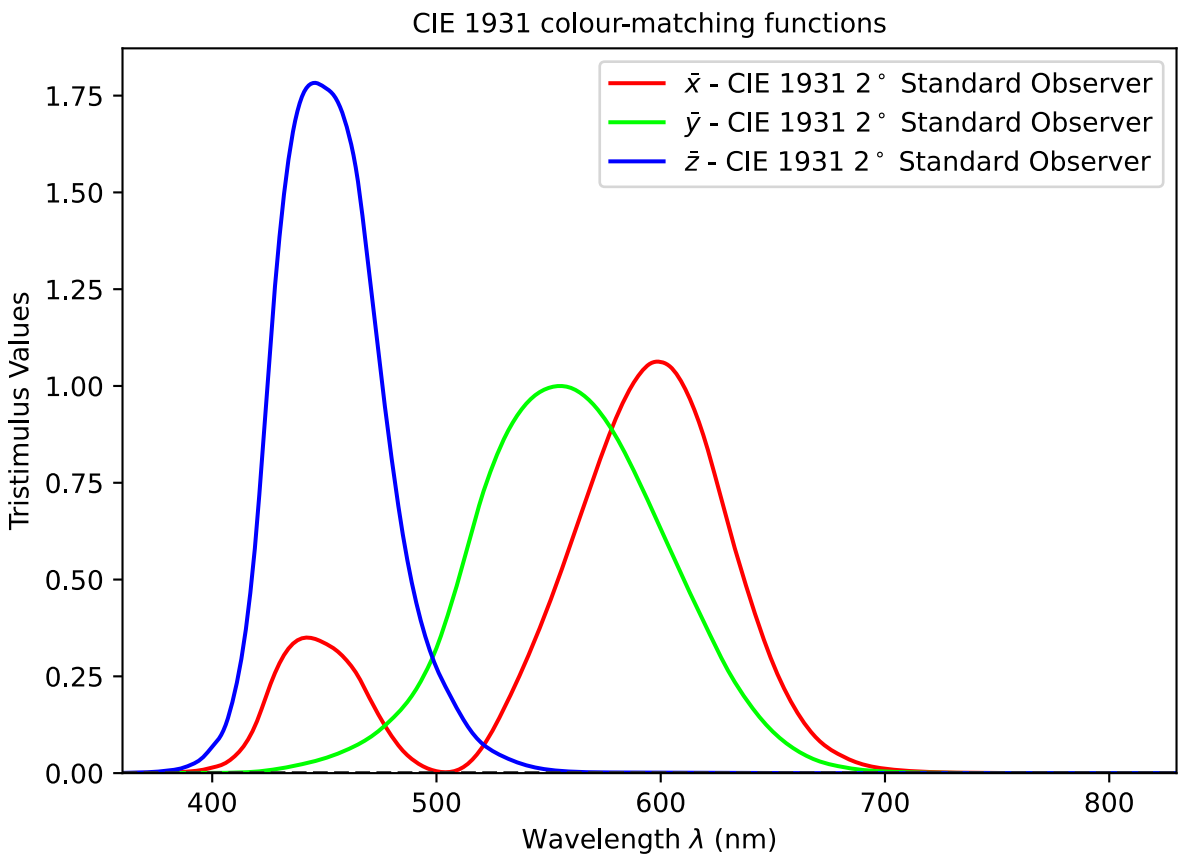


Figure 4.1.6 (reproduced). *The CIE 1931 color-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$. Each curve gives how much of one CIE primary is needed to match a unit of monochromatic light at that wavelength; integrating a spectrum against them yields the tristimulus values X, Y, Z. The $\bar{y}$ curve is, by construction, the luminance-sensitivity curve, so Y is luminance. Note that $\bar{x}$ dips negative in the cyan region, a fingerprint of non-orthogonal, non-negative color.*

LMS coordinates can be defined as a linear transform of XYZ for a chosen colorimetric observer. That does not make modern physiological cone fundamentals an exact transform of the older 1931 observer. For example, the Hunt–Pointer–Estévez transform used in appearance models is

$$\mathbf{LMS}_{\mathrm{HPE}} = \begin{pmatrix} 0.38971 & 0.68898 & -0.07868 \\ -0.22981 & 1.18340 & 0.04641 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{XYZ}. \tag{10}$$

These model coordinates should not be substituted silently for measured cone excitations or a different normalization of cone fundamentals.

💡 **BIG LESSON (L4.3) — LINEAR TRISTIMULUS COORDINATES ARE A MATRIX APART**

XYZ, a compatible LMS system, and linear RGB are different coordinates for a fixed observer's three-dimensional color matches. Nonlinear RGB encodings and perceptual spaces add specified nonlinear transforms. A real camera's raw measurements usually require an approximate color correction; an appearance model also needs viewing conditions.

Sidebar 8: Dataset: X-Rite ColorChecker (24-patch)

A reference chart used in color characterization. Its patches sample a useful set of surface colors but do not prove accurate reproduction of every spectrum. See Chapter 45.16, Datasets.

The negative-primary case is easiest to understand at the apparatus. Suppose a saturated test cannot be matched by adding any nonnegative mixture of the three primary lamps. Move some red primary light to the test side, then adjust green and blue on the matching side until the fields agree. The recorded red coefficient of the original test is negative because that red was added to the opposite side of the equality. No lamp emits negative light. Linearity lets us subtract its contribution algebraically.

Repeat the match with narrow-band test lights across wavelength. The three settings at each wavelength form the color-matching functions. A broad spectrum can then be treated as a sum of these narrow-band contributions, which is why the integral above predicts its tristimulus coordinates. Transforming those coordinates to XYZ changes the coordinate system; it does not require building three imaginary lamps. A monitor illustration can show the controls and equations, but its own primaries limit which test spectra and matches it can physically reproduce.

One might wish that color technology had started with LMS: its axes have a direct interpretation in terms of the receptors. Later measurements of cone photopigments and the development of cone fundamentals made that interpretation much more accessible [@bowmaker-dartnall-1980; @stockman-sharpe-2000]. By then XYZ was already embedded in instruments, specifications, and device characterization. Replacing the common reference would have imposed a compatibility cost on all of them. XYZ also supplies a convenient luminance coordinate and nonnegative matching functions. Its persistence is therefore understandable even though its imaginary primaries and experimental origins make it harder to explain.

Keep the historical argument separate from the observer model. Linear coordinate changes preserve the matches of a fixed observer. A later physiological observer can differ from the 1931 standard; adopting it is not merely renaming the older coordinates. We can explain why XYZ stayed in use without claiming that every modern cone fundamental is an exact transform of the 1931 tables.

Keeping track of what the numbers mean

in the book ↗

An array needs more than a label saying "RGB." Record its primaries, transfer function, white reference, numeric range, and whether it represents scene light or a rendered image. Relative XYZ also needs its normalization. A camera-native RGB array needs a camera characterization.

Before a transform, check these assumptions against its input. A matrix defined for linear RGB cannot consume sRGB-encoded samples directly. Assigning a profile declares how existing numbers should be interpreted; converting to another profile changes the numbers to preserve the intended color. White adaptation changes a reference condition and must not be applied twice.

I find this bookkeeping more troublesome than most of the individual equations. Several independent choices travel under one name such as “sRGB,” and applications often hide which choices they made. Explicit metadata and a few labeled intermediate arrays are more useful than guessing from whether the picture looks plausible.

Treat a color transform like a units check. A matrix labeled as taking linear Rec. 709 RGB to XYZ cannot be fed the encoded samples read from an sRGB image. A value labeled Y′ is luma, so substituting it for linear luminance in a physical-light calculation changes the problem. A transform expecting a D65 reference cannot silently be given D50-relative coordinates. Writing the encoding and white reference next to an intermediate array makes these errors visible before the picture is displayed.

There are two kinds of difficulty here. Some come from the measurement: three responses do not determine a spectrum, and the same surface sends different light under different illumination. Others come from conventions maintained across decades, such as print workflows using D50, common display spaces using D65, and the name sRGB covering both primaries and a transfer function. Separating the physical ambiguity from the bookkeeping helps identify what a better calibration can solve and what merely needs an explicit conversion.

A cleaner interface would separate the measurement of the incoming light from the choice of how a particular device should reproduce it. The measurement would identify its observer and normalization; the rendering would specify the viewing conditions, output device, and intended appearance. Color management tries to provide this separation through documented profiles and a common connection space (Chapter 4.8, Recap: the full color pipeline, color management and ICC). It has to work with existing representations, though. A name such as sRGB specifies primaries, a white reference, and an encoding together. Separating those choices remains work for the programmer. The type annotations make that separation explicit on the page.

Linear RGB

in the book ↗

LMS and XYZ are analysis spectra, built to *measure* color. To *store and reproduce* it we use the RGB family, and the standard member is sRGB (Stokes et al. 1996 (sRGB)), whose primaries are the Rec. 709 primaries under the D65 white point. In **linear light**, sRGB and XYZ are one 3×3 apart. The forward matrix takes a measured color into linear sRGB,

$$M : \mathrm{XYZ}_{[\mathrm{D65}]} \to \mathrm{RGB}_{709}, \qquad \begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.2040 & 1.0570 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \tag{12}$$

and its inverse recovers XYZ,

$$M^{-1} : \mathrm{RGB}_{709} \to \mathrm{XYZ}_{[\mathrm{D65}]}, \qquad \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}. \tag{13}$$

The coefficients shown are rounded. Apply them to linear values; a typical sRGB file must first be decoded. The middle row of the RGB-to-XYZ matrix gives relative luminance. Applying those weights to encoded components gives luma, which is neither luminance nor generally the encoding of luminance.

Negative RGB components indicate a target outside the positive cone spanned by these primaries. Components above one exceed the chosen reference range; they can represent greater intensity at an otherwise reproducible chromaticity. Floating-point scene-linear images often retain such values. Clipping to $[0, 1]$ is an output decision that can change hue and discard highlight information.

RGB color spaces

in the book ↗

An RGB space chooses primary chromaticities and a reference white; its stored representation also specifies a transfer function. Different choices serve different devices and workflows. sRGB is a common SDR delivery space. Display P3 extends its red and green coverage. Adobe RGB extends especially into greens and cyans useful in some print workflows. Rec. 2020 has still wider primary chromaticities. ProPhoto is a large working space with imaginary primaries.

With fixed integer precision, a larger gamut generally spreads codes more widely in color. The effect varies across the gamut and depends on the transfer function. Use sufficient precision for editing, then choose the delivery space from the output requirements. A screen cannot display every working-space value merely because software can store it.

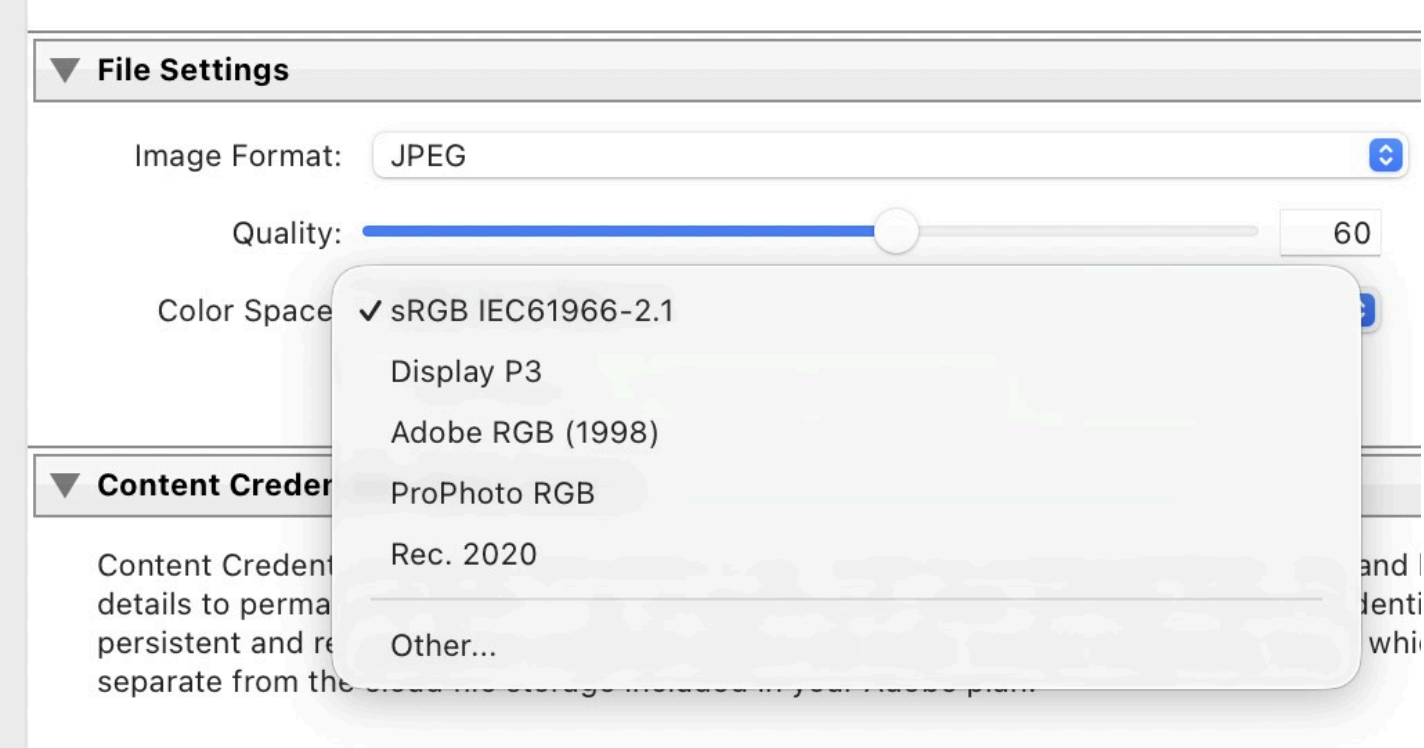


Figure 4.4.1 (reproduced). *The choice, as a photographer meets it. Lightroom's export panel asks for a color space alongside the file format and the JPEG quality, and the menu is the zoo of this chapter: sRGB for the web and for anyone whose screen you cannot know, Display P3 and Adobe RGB when the destination is wider, ProPhoto RGB and Rec. 2020 for editing and for delivery to displays that do not exist yet. Nothing in the pixel values says which one was meant, which is why the choice has to be recorded in the file and why getting it wrong shows up as color that is merely plausible rather than right. Screenshot of Adobe Lightroom Classic.*

> 💡 **BIG LESSON (L4.8) — THERE IS NO SINGLE RGB: GAMUT VERSUS BIT BUDGET**
>
> Choose a working gamut large enough for the content and operations, with sufficient numeric precision. Choose a delivery space the receiving system can interpret. Wider primaries alone do not improve an image, and converting an sRGB image to a wider space does not recover colors previously clipped.

Display-oriented spaces describe a reference set of primaries. Working spaces can instead choose imaginary primaries to enclose more physical colors. ProPhoto's green and blue are outside the spectral locus; its red is not the imaginary pair's other member. Such coordinates are useful for calculation, but their nonnegative cube contains values no physical light can produce.

Camera raw is different. Its channels are spectral measurements, often one color sample per photosite in a mosaic, not weights of three emission primaries. A camera would admit an exact universal XYZ matrix if its sensitivities spanned the standard observer's matching functions. Real cameras generally only approximate this condition, so color correction is fitted and depends on spectra and illumination. It is not meaningful to invert an arbitrary three-by-many sensitivity matrix and call the result three physical display primaries.

Camera profiles can provide multiple illuminant calibrations. Do not assume every DNG contains exactly two matrices at fixed temperatures; use the tags and conventions of the actual profile. Camera color-space menu settings usually govern rendered output and previews, while raw channel interpretation remains camera-specific.

The correction matrix also propagates noise. With equal independent input variances σ^2, output channel i has variance $\sigma^2 \sum_j M_{ij}^2$. More generally use $M\Sigma M^T$. Negative coefficients do not alone prove amplification; their magnitudes and the input covariance matter. The filter design and color fit balance sensitivity, color accuracy, and noise, as developed in Chapter 4.6, Sensing color: multiplexing strategies.

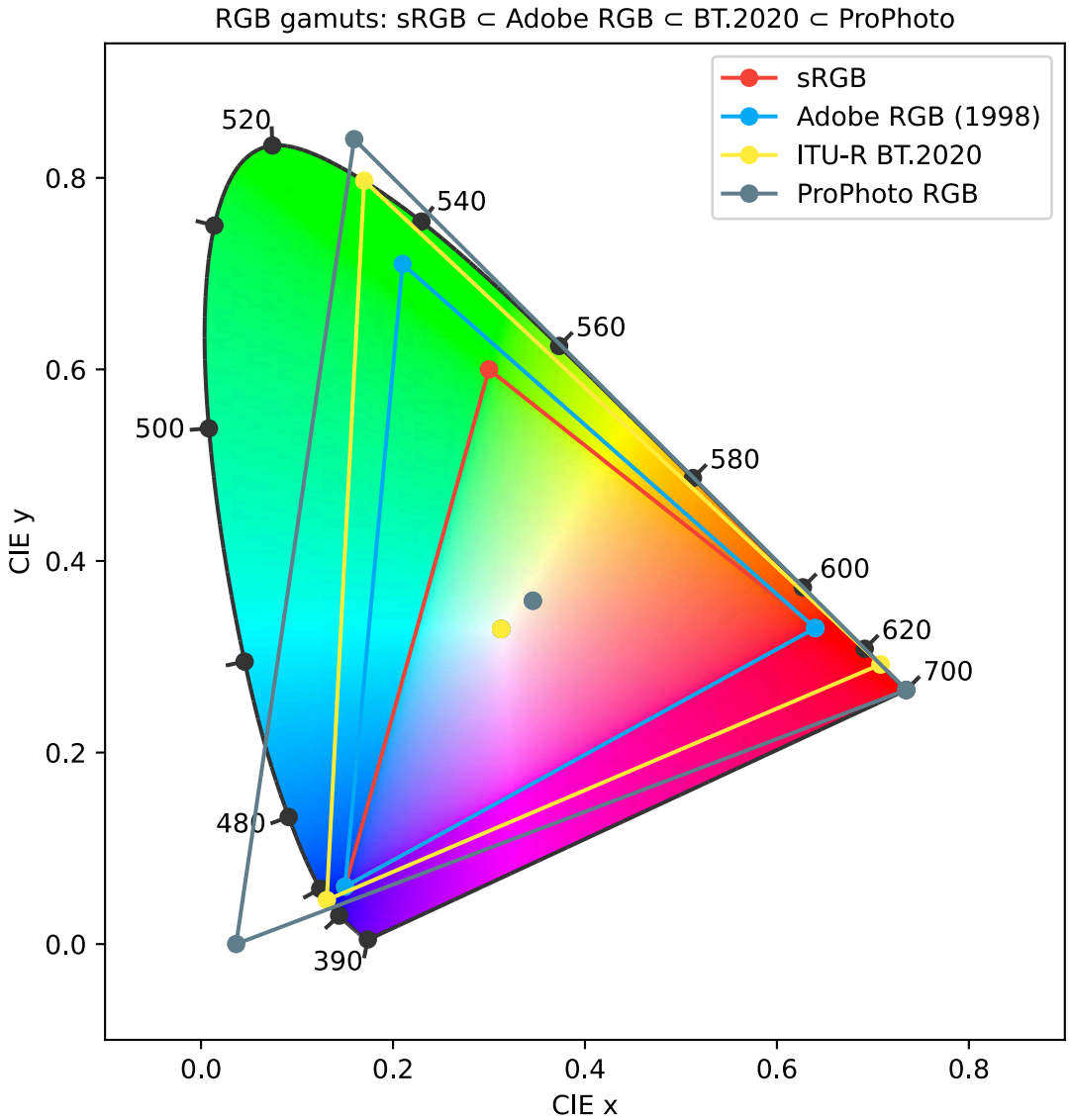


Figure 4.4.2 (reproduced). *RGB primaries on the chromaticity diagram. The triangle vertices are the red, green, and blue primaries of each space, and the triangle they enclose is that space's gamut. sRGB / Rec. 709 is the small triangle, and Adobe RGB, Rec. 2020 and ProPhoto enclose progressively larger areas: each really does contain the one before it, primary by primary. ProPhoto is the one to look at twice. Two of its three primaries lie* outside *the spectral locus, so they are not colors at all, and a corner of its triangle names nothing that can be seen. A change of space is a* 3×3 *matrix between these triangles.*

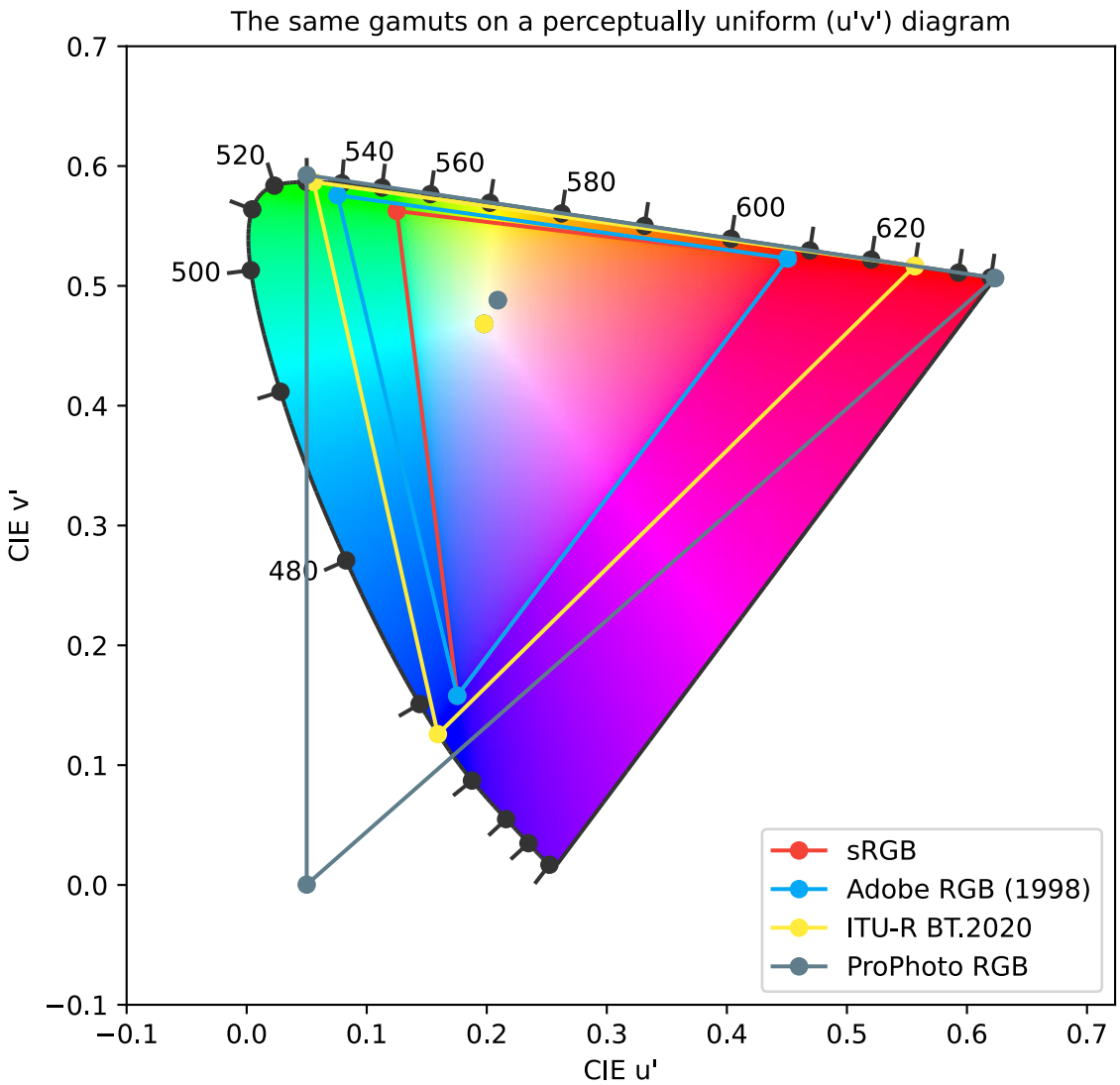


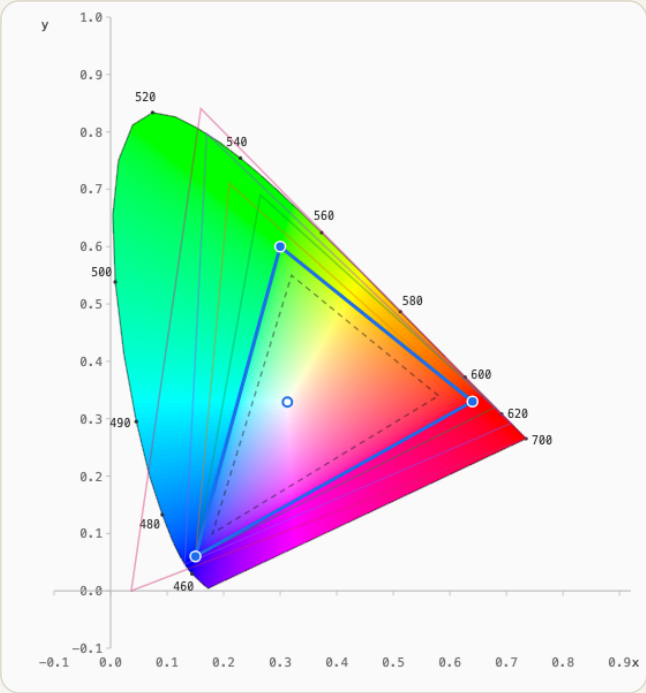
Figure 4.4.3 (reproduced). *The same RGB gamuts on a perceptually uniform diagram. The identical sRGB, Adobe RGB, BT.2020 and ProPhoto primaries of the previous figure, replotted on the CIE 1976 UCS ($u'v'$) chromaticity diagram, which spaces colors far more evenly by perceived difference. Compared with the xy diagram, the oversized green lobe collapses and the blues open up, so the triangles take on very different shapes and relative areas, from the very same numbers. The lesson: xy (and XYZ) measures color, it does not tell you how different two colors look. Rendered by colour-science.*

SPACE

sRGB / Rec.709 | Adobe RGB (1998) | Display P3 | Rec. 2020 | ProPhoto RGB | editable

reset editable

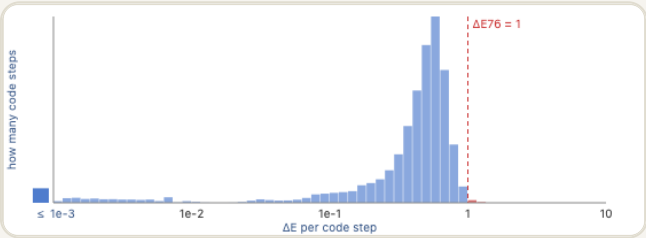
DIAGRAM ☐ more uniform (CIE 1976 *u′v′*)



sRGB / Rec.709: 33.3% of the horseshoe, 1.00x sRGB

QUANTIZATION power γ 2.2

bits 8



sRGB / Rec.709 at 8 bits, power γ 2.2 — median step 0.492 ΔE, 90th percentile 0.709, 0.3% of sampled steps over ΔE76 = 1

pure-power code range, dimmest non-black code to white: 1:2.0e+5, 17.6 stops

Quantization model: a pure power curve for every

the colors themselves, with every pixel the selected space cannot hold flagged

2.4% of pixels are outside sRGB / Rec.709 — a measured reflectance cube rendered under D65 (CAVE, Columbia)

gamut, not each format's specified transfer function. Actual 8-bit sRGB spans about 11.7 stops from code 1 to white. ΔE76 is a rough visibility proxy; 1 is not a universal detection threshold.

Figure 4.4.4 (reproduced). *Chromaticity and color spaces, interactive.* Every gamut is drawn at once on the CIE horseshoe, faintly, and one is **selected** from the row of buttons: that one is drawn in full, carries the draggable handles, and drives everything else. Selecting it also sets the encoding that space is really used with, sRGB, Adobe RGB and Display P3 at $\gamma \approx 2.2$ in 8 bits, Rec. 2020 at 2.4 in 10, ProPhoto at 1.8 in 16; both sliders stay free afterwards. The axes run a little below zero so a primary can be dragged *outside* the locus, which is where two of ProPhoto's live. A checkbox swaps xy for the perceptually uniform $u'v'$ diagram, and the coverage readout is measured in $u'v'$ either way: sRGB holds **33.3%** of the visible chromaticities, Adobe RGB 38.8% (1.17× sRGB), Rec. 2020 57.4% (1.72×), ProPhoto 76.7% (2.30×), and the editable triangle live as you drag it. **Below the diagram, what quantization costs in that space.** At every code in the space's own cube, step one level along R, G and B, decode both ends through that space's transfer and that space's primaries, and measure the step as a full CIELAB ΔE, lightness and chroma together and not brightness alone; the histogram is the distribution of those steps, and 1 ΔE serves as a rough proxy for a just-noticeable difference, which in practice varies with the color and the viewing conditions. Its axis is **fixed**, 0.001 to 10 ΔE, so two spaces can be compared by eye rather than by reading the ticks; steps past either end are counted into a bar outside the axis instead of being squeezed in. It answers to all three knobs. Widen the primaries and the same codes have more color to cover: at 8 bits and $\gamma = 2.2$, **0.3%** of sRGB's code steps are visible, against 8.0% for Rec. 2020 and **23.3%** for ProPhoto, which is exactly why ProPhoto is specified at 16, where it is back to 0.0%. Change γ instead, in sRGB at 8 bits, and you get the other half of the story: 6.5% of steps visible at $\gamma = 1$, 0.3% at 2.2, none at 3.0, while the *median* step grows. Gamma does not remove the error, it levels it. Underneath, the **dynamic range** those same codes buy, from the dimmest non-black code up to white: 8.0 stops at $\gamma = 1$, which is one stop per bit and no more, against 17.6 at $\gamma = 2.2$ in the same 8 bits, 24.0 for Rec. 2020's 10 bits at 2.4, and 28.8 for ProPhoto's 16 at 1.8. Unlike the histogram, that number is set by the encoding alone and not by the primaries, since every space here puts white at $Y = 1$. Read it as an upper bound: the slider is a *pure* power, whose slope at zero is infinite, so it reaches further into the dark than a real transfer curve with a linear toe. sRGB's actual piecewise curve gives 11.7 stops at 8 bits, not 17.6. **On the right**, one sample throughout, a measured reflectance cube rendered under D65, with every pixel the selected space cannot hold flagged. Only 2.4% of these paints leave sRGB, 0.6% leave Display P3 and none leaves Rec. 2020, which is this section's argument in three numbers: real pigments barely need more than the oldest gamut.

This distinction explains two different kinds of color-conversion experiment. Between linear RGB working spaces built from the same standard observer, convert a color to XYZ and then to the destination coordinates; the two matrices compose into one exact coordinate transform. For a camera, photograph a set of known reflectances under a specified light and fit a matrix from the measured camera responses to their target XYZ values. It can fit those samples well and still make errors on other spectra. Two spectra that the camera records identically cannot be separated by any later matrix, even if the standard observer distinguishes them. More training patches improve the fit over a chosen population; they do not restore spectral information absent from the three measurements.

The choice of a working gamut also affects quantization. With a fixed integer bit depth, a wider set of primaries spreads the available code combinations over a larger range of colors. It can retain a saturated color that a smaller space clips, yet give coarser steps in a region shared by both spaces. Increasing bit depth or using floating point reduces that pressure. This is why a space suited to intermediate editing need not be the best choice for compact

delivery. The working space must retain the colors the pipeline needs; the output space must match the interpretation supported by the destination.

A caution about that diagram before we read too much into the triangle sizes: the xy chromaticity plane is a space for *measuring* color, not for judging how different two colors look. It stretches the greens enormously and cramps the blues and purples, so neither the relative areas of these gamuts nor the distances between their primaries track perceived difference. Re-plot the *identical* primaries on a perceptually more uniform diagram, the CIE 1976 UCS ($u'v'$) space, and every triangle changes shape (Figure 4.4.3): the relative areas and distances change. The newer diagram is more uniform, but it is still not a universal perceptual ruler. That XYZ (and its xy projection) is *not* perceptually uniform is precisely what will force the deliberately non-linear spaces of Chapter 4.5, Advanced color: opponents, perceptually-uniform, and preferred.

Two design processes can lead to an RGB space. In a display-oriented design, begin with the primaries a reference display can emit, then describe colors as amounts of those lights. In a working-space design, begin with the colors that editing should retain, then choose a coordinate system that encloses them. The latter need not correspond to a manufacturable display. An imaginary primary is acceptable as a coordinate axis because no lamp is being asked to emit it while the image is stored or edited. Physical realizability becomes an output constraint when those values are converted for a screen or printer.

This distinction explains why choosing a larger working space does not upgrade the monitor. It changes which intermediate colors can be represented without clipping to the working cube. Displaying them still requires conversion and, where needed, gamut mapping. Conversely, a delivery space can be chosen for compatibility with the intended viewing path even when a larger space was useful during editing. The export dialog asks for an engineering choice about that destination, not a ranking in which the largest triangle is always best.

The noise formula can be read numerically. A correction row with coefficients 1.8, -0.9, and 0.1 has squared length 4.06. With equal independent input noise, its output standard deviation is therefore about 2.01 times the standard deviation of one input channel. The row may improve the color fit while increasing that channel's noise. This is why a camera characterization should be evaluated for both reconstruction error and noise propagation, using the actual covariance rather than assuming that every color matrix has the same cost.

Reference: primaries, matrices, and transfer functions

OPTIONAL
in the book ↗

Each space named above is fixed by two things: its primaries and white point, which set the linear RGB-to-XYZ matrix, and its transfer function, the gamma curve that encodes the linear values for storage. The matrices below carry *linear* light, no diacritic, into XYZ; the transfer is applied afterward to get the stored $R'G'B'$ (the primes of Chapter 4.3, Encoding: linear, gamma or log). The middle row of each matrix is that space's luminance Y weighting, the coefficients also used, with the specified nonlinear encoding, to define a corresponding luma (L4.5 show).

sRGB and Rec. 709 primaries (D65) share this linear matrix, though their transfer conventions differ:

$$M_{\text{sRGB}} : \text{RGB}_{[\text{D65}]} \to \text{XYZ} = \begin{pmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{pmatrix} \tag{14}$$

sRGB transfer: piecewise, a linear segment $R' = 12.92\,R$ for $R \leq 0.0031308$ splicing into $R' = 1.055\,R^{1/2.4} - 0.055$, overall close to $\gamma = 2.2$.

Adobe RGB (1998) (D65), wider greens for print work:

$$M_{\text{Adobe}} = \begin{pmatrix} 0.5767 & 0.1856 & 0.1882 \\ 0.2974 & 0.6274 & 0.0753 \\ 0.0270 & 0.0707 & 0.9911 \end{pmatrix} \tag{15}$$

Transfer: a pure power law, $\gamma = 2.19921875$ (that is $563/256$).

Display P3 (D65), the DCI-P3 primaries on the sRGB white:

$$M_{\text{P3}} = \begin{pmatrix} 0.4866 & 0.2657 & 0.1982 \\ 0.2290 & 0.6917 & 0.0793 \\ 0.0000 & 0.0451 & 1.0439 \end{pmatrix} \tag{16}$$

Transfer: the sRGB piecewise curve.

ITU-R BT.2020 / Rec. 2020 (D65), ultra-high-definition with very wide primaries (its red is a near-monochromatic 630 nm):

$$M_{2020} = \begin{pmatrix} 0.6370 & 0.1446 & 0.1689 \\ 0.2627 & 0.6780 & 0.0593 \\ 0.0000 & 0.0281 & 1.0610 \end{pmatrix} \tag{17}$$

Transfer: BT.2020 specifies a camera-side piecewise encoding with a 0.45-power segment and linear toe. Specify the display response separately. Rec. 2100 uses the same primary chromaticities with PQ or HLG HDR systems (Chapter 4.3, Encoding: linear, gamma or log).

ProPhoto RGB (ROMM) (white point D50), a large editing space with imaginary green and blue primaries:

$$M_{\text{ProPhoto}} : \text{RGB}_{[\text{D50}]} \to \text{XYZ} = \begin{pmatrix} 0.7977 & 0.1352 & 0.0313 \\ 0.2880 & 0.7119 & 0.0001 \\ 0.0000 & 0.0000 & 0.8249 \end{pmatrix} \tag{18}$$

Transfer: $\gamma = 1.8$, with a short linear segment (slope 16) below $R = 1/512$.

Two reminders the type system enforces. The matrices are all defined on *linear* light, so a conversion between two of these spaces is $M_2^{-1} M_1$ applied to linear RGB, never to the stored $R'G'B'$: decode first. And the white points are not all the same (ProPhoto is D50, the rest D65), so a white-relative ProPhoto-to-sRGB conversion includes chromatic adaptation (often Bradford, Chapter 4.2, White balance and white point) between the XYZ stages. A stimulus-preserving conversion has a different objective and does not silently adapt the white.

For a white-relative conversion, the full order is

```
linear_source = decode_source(encoded_source)
XYZ_source = M_source @ linear_source
XYZ_target = adapt_white(XYZ_source, source_white, target_white)
linear_target = solve(M_target, XYZ_target)
# Retain extended range while working; map gamut for the chosen output.
encoded_target = encode_target(mapped_linear_target)
```

The printed matrices are rounded reference values. Use a consistent precision and white normalization in code. Verify that RGB white maps to the stated XYZ white and that an unmodified in-gamut round trip returns its input.

Sensing color: multiplexing strategies

Synthesizing color

OPTIONAL
in the book ↗

Sensing a color projects a spectrum onto three numbers; synthesizing one runs that backward, building light from a few primaries so that a human sees the intended color. The projection threw information away and cannot be undone, so synthesis does not rebuild the original spectrum, only a metamer of it, and it must do so with real lights or real inks, which can only *add* or *absorb*, never contribute a negative amount. That single constraint, non-negativity, shapes everything in this chapter: which colors a device can reach (its gamut), what has to happen to the ones it cannot (gamut mapping), and how displays, printers, and film each make their compromise with it.

Color TLDR

Physical color

in the book ↗

Before any eye or sensor, color is *physical*: a light is a **spectrum**, its power at each wavelength (the spectral power distribution). For a surface, the light that reaches the eye is not the surface's own but the illumination *times* what the surface reflects, $C(\lambda) = S(\lambda)\,\rho(\lambda)$: the illuminant spectrum $S(\lambda)$ multiplied wavelength by wavelength by the surface reflectance $\rho(\lambda)$. An apple looks red because its reflectance is high in the reds and low elsewhere, but only when there is red light to reflect: under a pure green lamp the same apple is nearly black. Self-luminous things (a screen, a flame, the sky) skip the product and emit their spectrum directly. Everything after this reduces that spectrum to a few numbers. Details: Chapter 4.1, Linear color reproduction and CIE XYZ.

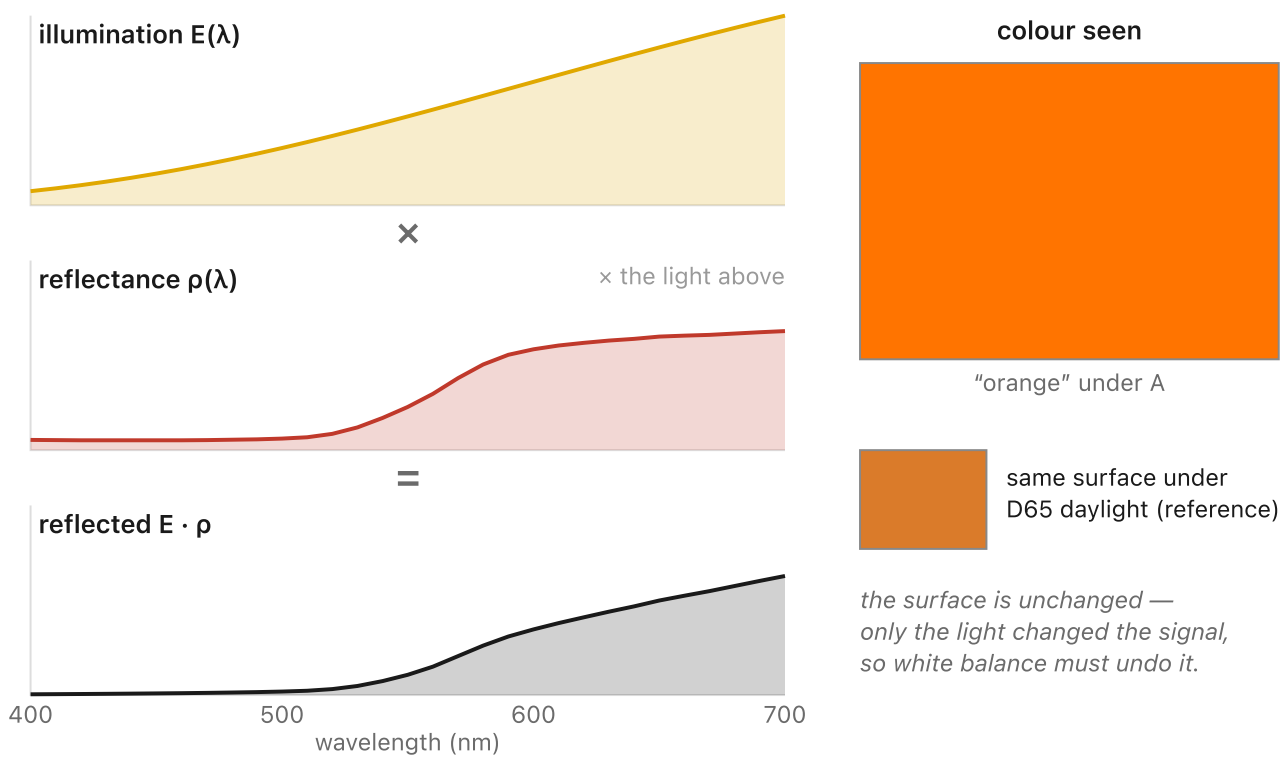


Figure 2.1.28 (reproduced). *Color is illumination × reflectance. A surface's fixed reflectance $\rho(\lambda)$ is multiplied wavelength by wavelength by the illuminant spectrum $S(\lambda)$ actually present, and the product $C(\lambda) = S(\lambda)\,\rho(\lambda)$ is the light that reaches the eye. Change the light and the stimulus changes even though the surface has not.*

Cones, LMS, and metamerism

in the book ↗

The eye has three types of color-sensitive photoreceptors. Given a physical spectrum, their response L, M, S is a projection (dot product) of the light onto three sensitivity curves. That is, it's the integral of the light spectrum times the sensitivity of each cone. Because the sensitivity curves overlap, the three axes, L, M, S, are **non-orthogonal** (there is cross-talk), and because light cannot be negative, not every point is reachable. And because only three numbers survive, many different spectra collapse to the same triple: **metamers**. That is, metameters are different physical spectra that project on the same L, M, S, and are therefore perceived as the same color. This is exactly what lets a three-primary screen reproduce any color. Details: Chapter 4.1, Color space 101.

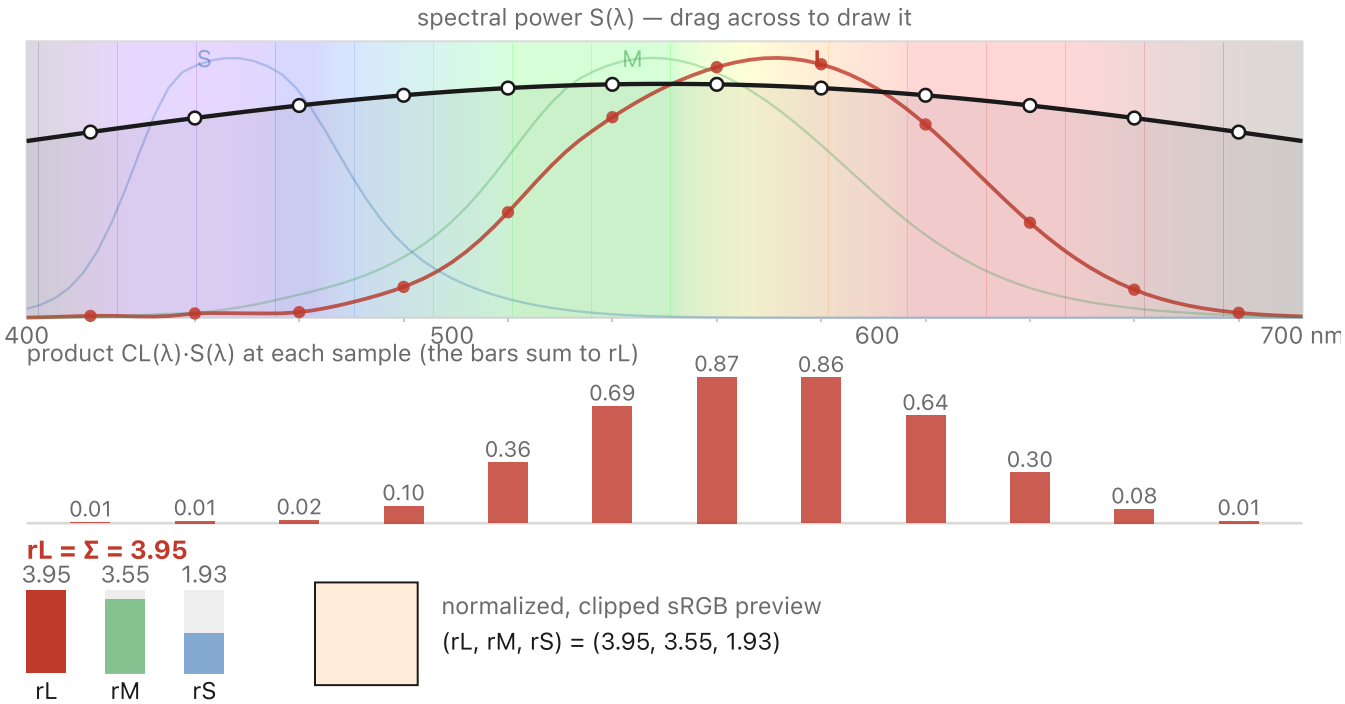


Figure 3.2.2 (reproduced). *The three cone sensitivities. L, M, and S response curves overlap across the visible band, so each cone response is the spectrum projected onto its curve.*

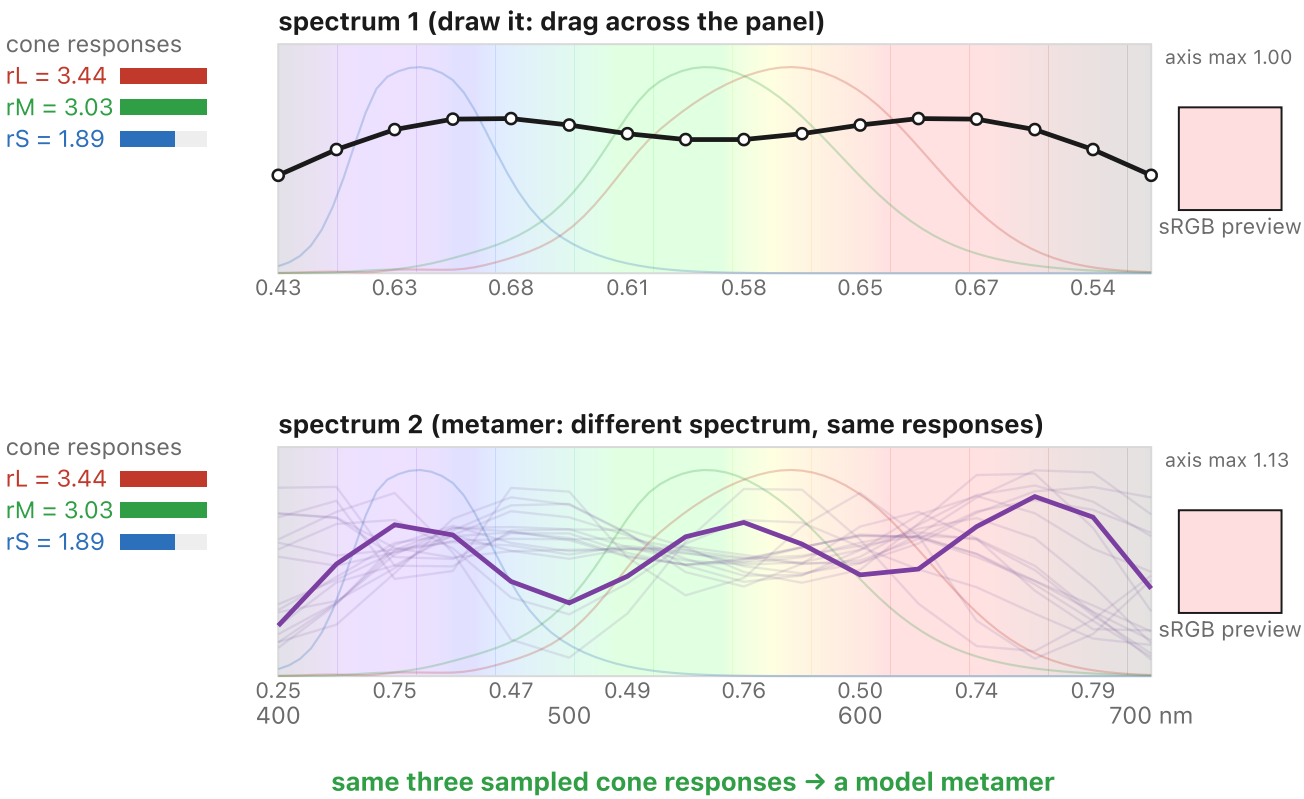


Figure 3.2.10 (reproduced). *Metamers. Two different spectra that integrate to the same (L, M, S) and so look identical, the ambiguity that lets three primaries reproduce any color.*

XYZ

in the book ↗

In an ideal world, color science would be built on L, M, S, but for historical reasons (it took a while to have technology to measure cone responses), the standard we all build on is **CIE XYZ**. It is a linear transform of LMS, one 3×3 matrix away; its coordinates are semi-arbitrary, with one exception, Y is defined to be **luminance**. It was originally built through color *reproduction* experiments rather than by measuring cones, but you do not need that history to use it: treat XYZ as the standard, device-independent coordinate system everyone converts through. Details: Chapter 4.1, Color space 101.

Chromaticity and gamut

in the book ↗

For visualization purposes, it is convenient to drop brightness and use a 2-D **chromaticity** diagram. The standard version starts with XYZ and divided by the sum of the three numbers. Physically realizable colors (spectra with no negative light) occupy a horseshoe shape with monochromatic light at the boundaty. A device's reproducible colors, the mix of its primaries, form a **convex** region (a triangle for three primaries) inside it: its **gamut**. Colors outside the triangle cannot be made and must be mapped in, which is called gamut mapping. Details: Chapter 4.1, Color space 101, Chapter 4.7, Synthesizing color.

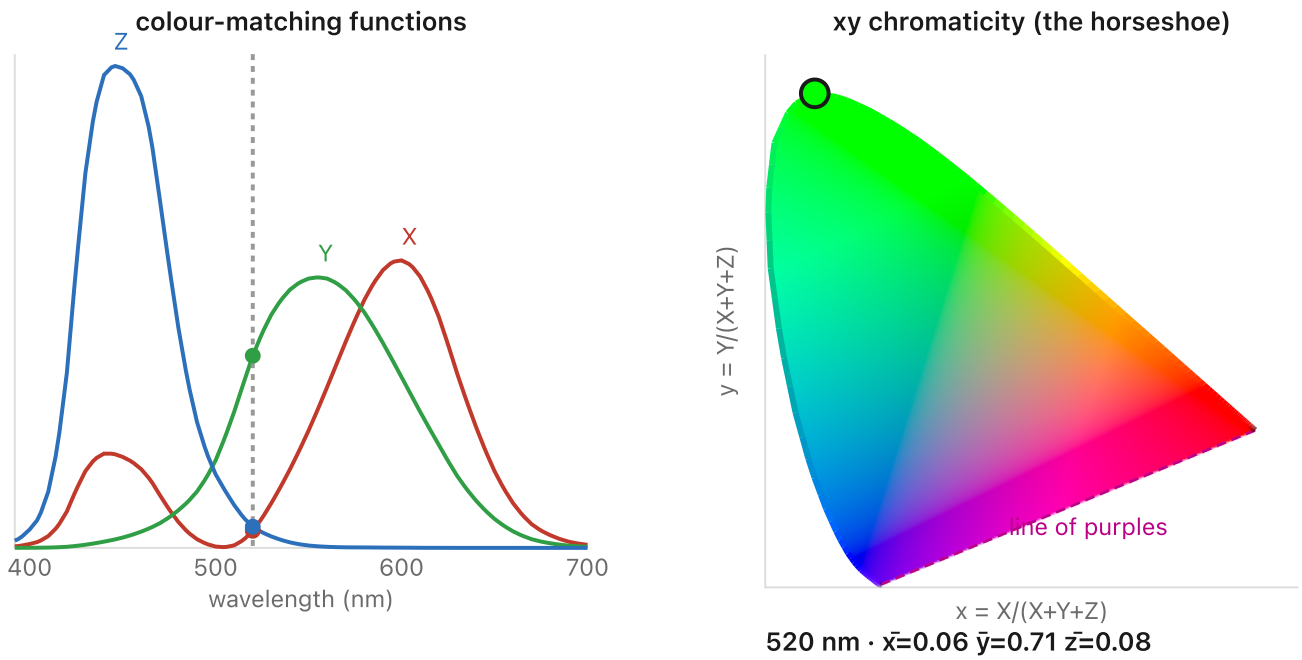


Figure 4.1.9 (reproduced). *The chromaticity horseshoe. Realizable colors fill the horseshoe, the spectral locus colored by wavelength with its interior the actual colors, and the white point near the center. It opens on the CIE xy view; switch to LMS or MacLeod-Boynton to see the same shape in cone coordinates.*

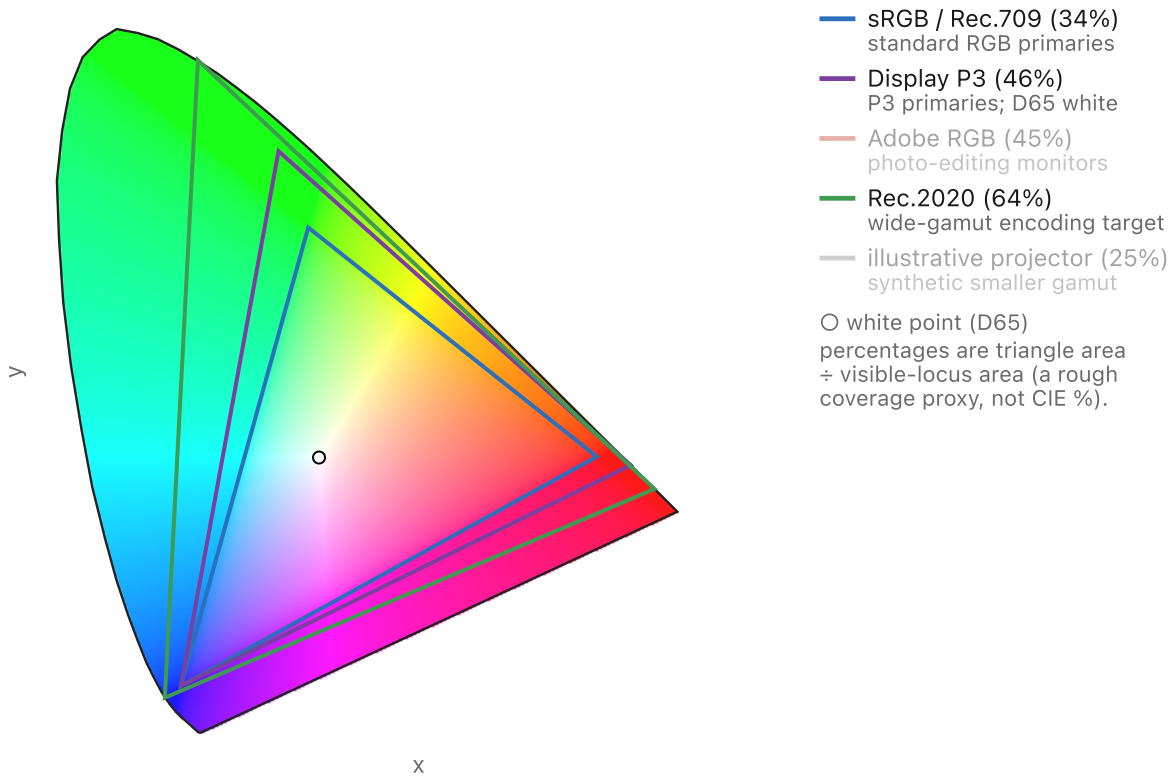


Figure 4.7.5 (reproduced). *Chromaticity and gamut. The horseshoe of realizable colors, with several device gamuts (sRGB, Adobe RGB, Display P3, Rec.2020) drawn as triangles inside it.*

Linear RGB, and why there are many

in the book ↗

There are *many* named RGBs (sRGB, Adobe RGB, Display P3, Rec.2020, ProPhoto). Getting to one RGB space from XYZ is again a single 3×3 matrix.

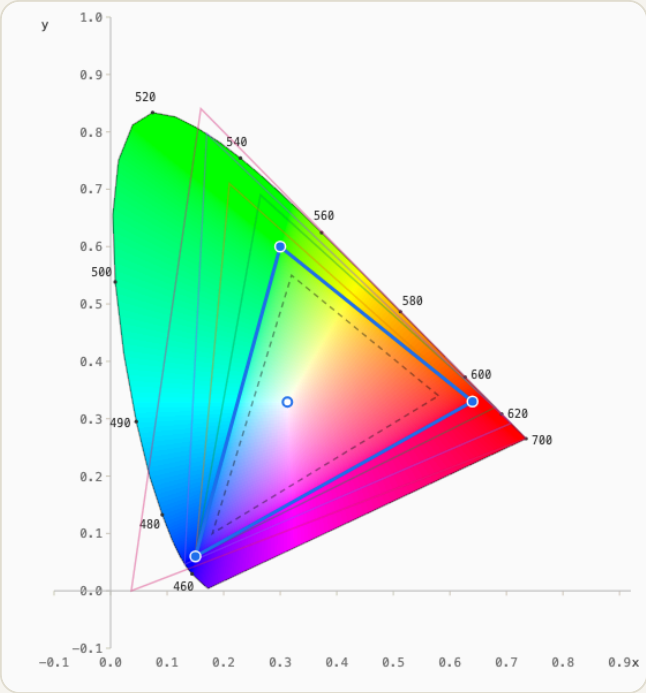
People define different RGBs for three practical reasons: different display **primaries** (so different gamuts), and different trade-offs of **bit budget and bandwidth** (a wider gamut spreads the same bits thinner). Pick by what you need to cover and how many bits you can spend. Details: Chapter 4.4, RGB color spaces.

SPACE

sRGB / Rec.709 | Adobe RGB (1998) | Display P3 | Rec. 2020 | ProPhoto RGB | editable

reset editable

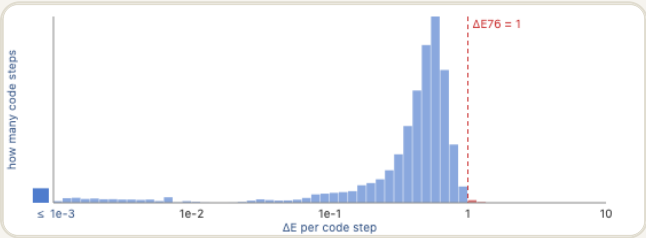
DIAGRAM ☐ more uniform (CIE 1976 *u′v′*)



sRGB / Rec.709: 33.3% of the horseshoe, 1.00x sRGB

QUANTIZATION power γ 2.2

bits 8



sRGB / Rec.709 at 8 bits, power γ 2.2 — median step 0.492 ΔE, 90th percentile 0.709, 0.3% of sampled steps over ΔE76 = 1

pure-power code range, dimmest non-black code to white: 1:2.0e+5, 17.6 stops

Quantization model: a pure power curve for every

the colors themselves, with every pixel the selected space cannot hold flagged

2.4% of pixels are outside sRGB / Rec.709 — a measured reflectance cube rendered under D65 (CAVE, Columbia)

gamut, not each format's specified transfer function. Actual 8-bit sRGB spans about 11.7 stops from code 1 to white. ΔE76 is a rough visibility proxy; 1 is not a universal detection threshold.

Figure 4.4.4 (reproduced). *Why there are many RGBs. Toggle the standard gamuts and drag the primaries and white point; a wider triangle reaches more colors but spreads the same code values thinner.*

Color terms

Radiometry: physical light, no observer

in the book ↗

Radiometry. The physical measurement of light as electromagnetic power, with no reference to a human observer.

Radiance. Power per unit projected area per unit solid angle, in $W \cdot sr^{-1} \cdot m^{-2}$. The fundamental quantity a camera measures, and the one preserved along a ray through empty space; sensor- and observer-independent. Written L (linear).

Irradiance. Power per unit area arriving on a surface, in $W \cdot m^{-2}$. What the sensor plane actually receives; it is tied to scene radiance through the lens by the aperture (f-number) and the projection geometry. Written E.

Spectral radiance. Radiance per wavelength interval, $L(\lambda)$, with units such as $W \cdot sr^{-1} \cdot m^{-2} \cdot nm^{-1}$. A **spectral power distribution (SPD)** describes power or another specified radiometric quantity versus wavelength; state which quantity and normalization. A spectrum alone does not specify the spatial, angular, temporal, or polarization distribution of light.

Photometry: physical light weighted by the eye

in the book ↗

Photometry. Radiometry weighted by the human luminosity function $V(\lambda)$, the daytime spectral sensitivity of vision. It defines standard observer-weighted physical quantities, not a complete prediction of perceived brightness. Other photometric regimes use other specified weightings.

Luminous flux. Total light power weighted by $V(\lambda)$, in lumens.

Luminance. The photometric counterpart of radiance: radiance weighted by $V(\lambda)$, in $cd \cdot m^{-2}$. A linear weighted integral of spectral radiance; it is not proportional to total photon count independently of spectrum. It correlates with brightness but is not itself a percept. Written Y in XYZ (or L where context is clear). Physical color mixing can operate on full linear RGB or spectra; luminance alone discards chromatic information.

Illuminance. The photometric counterpart of irradiance, the luminous flux arriving per unit area, in lux.

> The chronic confusion, luminance Y vs luma Y': Y is linear, photometric and device-independent; Y' is computed from specified nonlinear $R'G'B'$ and depends on the encoding and coefficients; it is generally not the encoding of Y. Same letter, different domain, different meaning (L4.5 show).

Colorimetry: color via the standard observer

in the book ↗

Colorimetry. The measurement of color, made repeatable by fixing a *standard observer* whose sensitivities stand in for a human's.

Color-matching functions (CMFs). $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$: the CIE 1931 standard observer's three sensitivity curves, measured empirically from color-matching experiments. They are the mechanism by which a spectrum is projected onto three numbers.

Tristimulus values. X, Y, Z: the three numbers from integrating the SPD against the CMFs over wavelength with a specified normalization. They are linear in the stimulus; Y is relative luminance or absolute luminance according to the scale.

Metamerism. Two different spectra that integrate to the *same* XYZ for a given observer, and match under the specified viewing conditions. Camera metamerism can differ from standard-observer metamerism. Surface matches can change with illumination, whether or not the illuminant is spiky.

Chromaticity. Tristimulus coordinates with overall scale removed: $x = X/(X+Y+Z)$, $y = Y/(X+Y+Z)$. A *projective* quantity, reduced from three dimensions to two, so it names a color's direction but not its amount and cannot be turned back into XYZ without Y supplied separately.

Chromaticity diagram. The 2-D plot of x, y (or the more uniform u', v') chromaticities, the horseshoe. Three-primary additive gamuts project to triangles and the blackbody illuminants as the Planckian locus.

Spectral locus. The curved boundary of the horseshoe, the chromaticities of monochromatic (single-wavelength) lights. For ordinary broadband illumination, a bounded surface reflectance approaching a spectral endpoint generally has a low maximum luminance.

Line of purples. The straight edge closing the horseshoe between the red and violet ends of the spectral locus. Its colors are real but have no single-wavelength equivalent; they are mixtures of the two spectral extremes.

Chronic confusions, named

in the book ↗

Nine pairs this part keeps apart, and the distinction in each:

term	vs	the distinction
luminance Y	luma Y'	linear photometric quantity vs weighted nonlinear components
lightness L^*	luminance Y	perceptual correlate vs linear photometric quantity with a stated scale
chroma C^*	C_b, C_r	perceptual magnitude in CIELAB vs encoding difference signals
saturation	chroma C^*	relative to the object's brightness vs relative to white
chromaticity x, y	color XYZ	projective 2-D (luminance discarded) vs full 3-D
white point (space)	white balance (camera)	a mathematical normalization vs illuminant compensation
OETF	EOTF	camera encoding vs display decoding
colorfulness	chroma C^*	absolute percept vs white-relative percept
primaries	phosphors / LEDs	abstract basis vectors vs physical emitters

One word, several meanings

in the book ↗

A few terms carry different meanings in different chapters and are worth disambiguating on sight:

White point has four senses: a color-space normalization parameter, a display's calibration target, a viewer's adaptation state, and a camera's auto-white-balance target. They are designed to agree in a calibrated pipeline but are conceptually independent.

Color temperature is the temperature (in kelvin) of the blackbody whose glow matches a light's color, so it is only exactly meaningful on the Planckian locus; it is used loosely for any illuminant's chromaticity, and as a camera white-balance control even for non-blackbody sources. Correlated color temperature specifies a nearby Planckian-locus temperature; it does not specify the source spectrum or its off-locus distance.

Neutral means achromatic under specified conditions. Equal RGB components represent the defined RGB white chromaticity; equal XYZ represents equal-energy white, not every neutral. Equal LMS is a normalization-dependent convention, not a general test of neutrality.

Linear can mean proportional to physical light (radiometrically linear), a linear-algebra operation (a matrix, no nonlinearity), or a 1-D sensor layout (a linear array). It always needs its context.

Recap: big lessons in this lecture

💡 **BIG LESSON (L2.1) — LIGHT IS A SPECTRAL DISTRIBUTION**

Light, and the physical color it carries, is described by a spectral distribution: the power it carries at each wavelength, a function $S(\lambda)$. One wavelength is a single pure color; real light is a weighted mix of all of them. Everything downstream (how a prism fans light out, how a surface reflects it, how a sensor or an eye turns it into a perceived color) starts from this per-wavelength power.

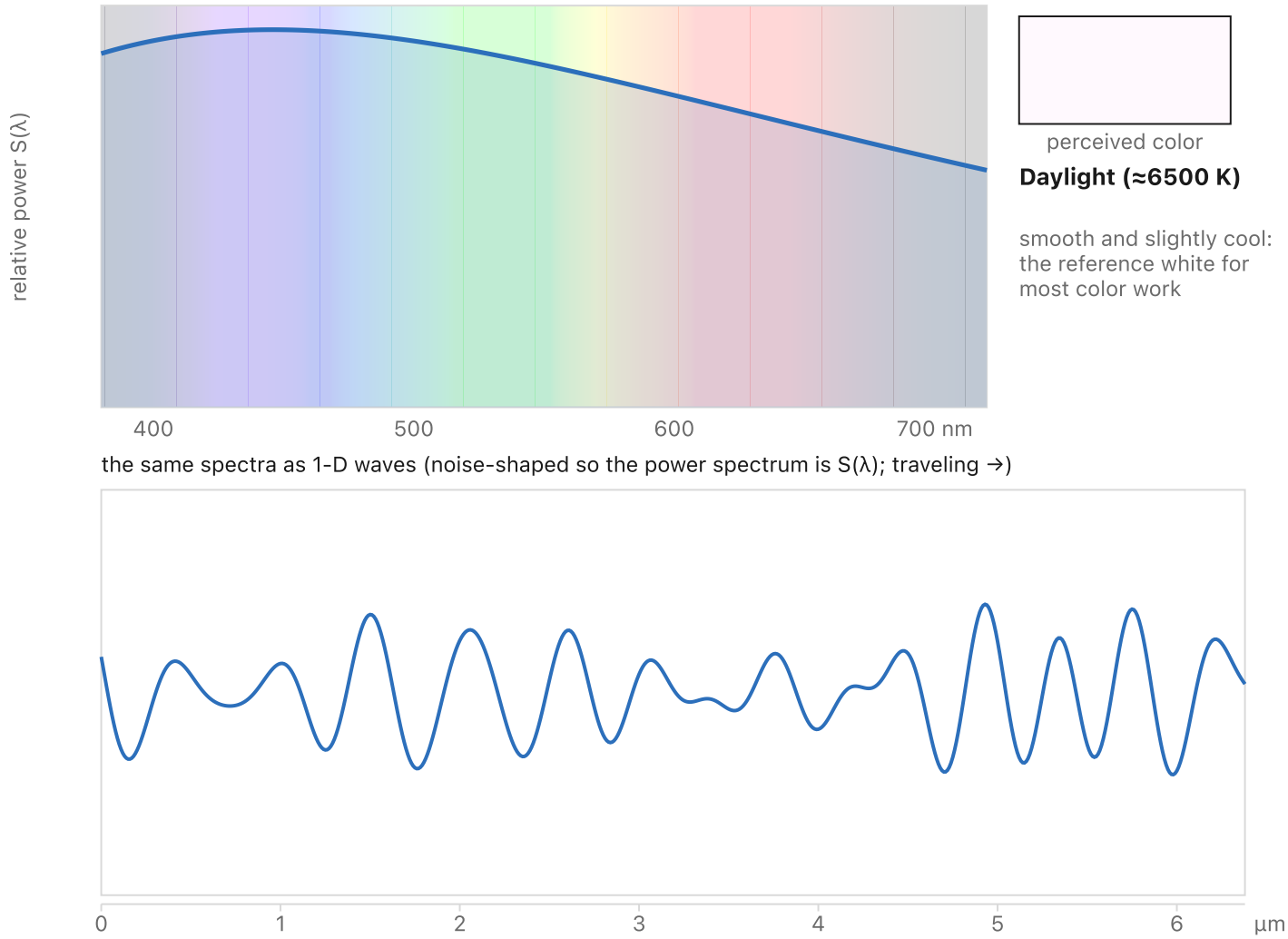


Figure 2.1.6 (reproduced). *The spectra of real light sources.* Each "white" source carries a different spectral power distribution $S(\lambda)$: daylight and tungsten are smooth Planck curves (cool and warm), a fluorescent tube adds sharp mercury lines on broad phosphor humps, a white LED is a blue spike feeding a yellow phosphor, and clear sky is daylight steepened by Rayleigh scattering. The swatch gives each source's perceived color; several read as nearly white despite very different spectra. *Interactive: pick a source, or overlay them all; each spectrum also plays as a 1-D traveling wave, and a bandpass-filter slider isolates a band to show the surviving wave and its color: a narrow band gives a clean tone and a saturated hue, a wide band stays noisy and washes toward white.*

💡 **BIG LESSON (L2.5) — EVERY MATERIAL DISPERSES**

The refractive index is not a constant: $n(\lambda)$ depends on wavelength: in ordinary glass it rises toward the blue, so blue bends more than red. Because of this you *cannot route every wavelength the same way with a single refraction*: a prism fans white light into a spectrum, a raindrop makes a rainbow, and a simple lens focuses blue and red at slightly different distances: **chromatic aberration**. Much of the craft of lens design (combining crown and flint glass into achromatic and apochromatic groups) exists to fight this one fact. Dispersion is not a defect of cheap glass; it is built into how matter and light interact.

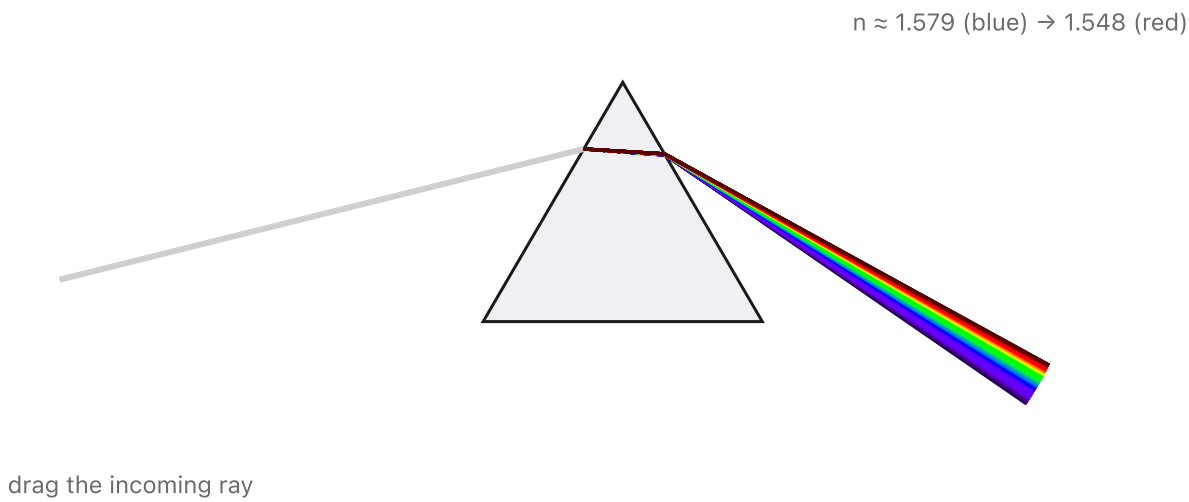


Figure 2.1.15 (reproduced). *Dispersion in a prism. Because the refractive index varies with wavelength (Cauchy's law: n higher toward the blue), each color bends by a slightly different amount at each glass face, so a single white ray entering the prism fans out into a spectrum. This is just Snell's law applied wavelength-by-wavelength, the same physics behind the rainbow and behind chromatic aberration in lenses. Drag the incoming ray, or change the glass index and dispersion.*

💡 **BIG LESSON (L2.6) — THE MULTIPLICATIVE WORLD**

Much of imaging is *multiplicative*, not additive. The light reaching the eye is illumination **times** reflectance; contrast is a **ratio** of intensities; a shadow darkens a region by a **factor**, not by subtracting a constant. Because the structure is multiplicative, the natural tools are **multiply, divide, and log**, not just add. This is why, again and again, we will work with ratios and with logarithmic encodings rather than raw linear values. It is also why **white balance** is hard: to recover an object's true color we must *divide out* the unknown illuminant $S(\lambda)$ from the product $C(\lambda)$, undoing a multiplication we never observed directly. (The additive counterpoint, light from different sources *adding*, and what it means for choosing an encoding, comes in Color technology.)

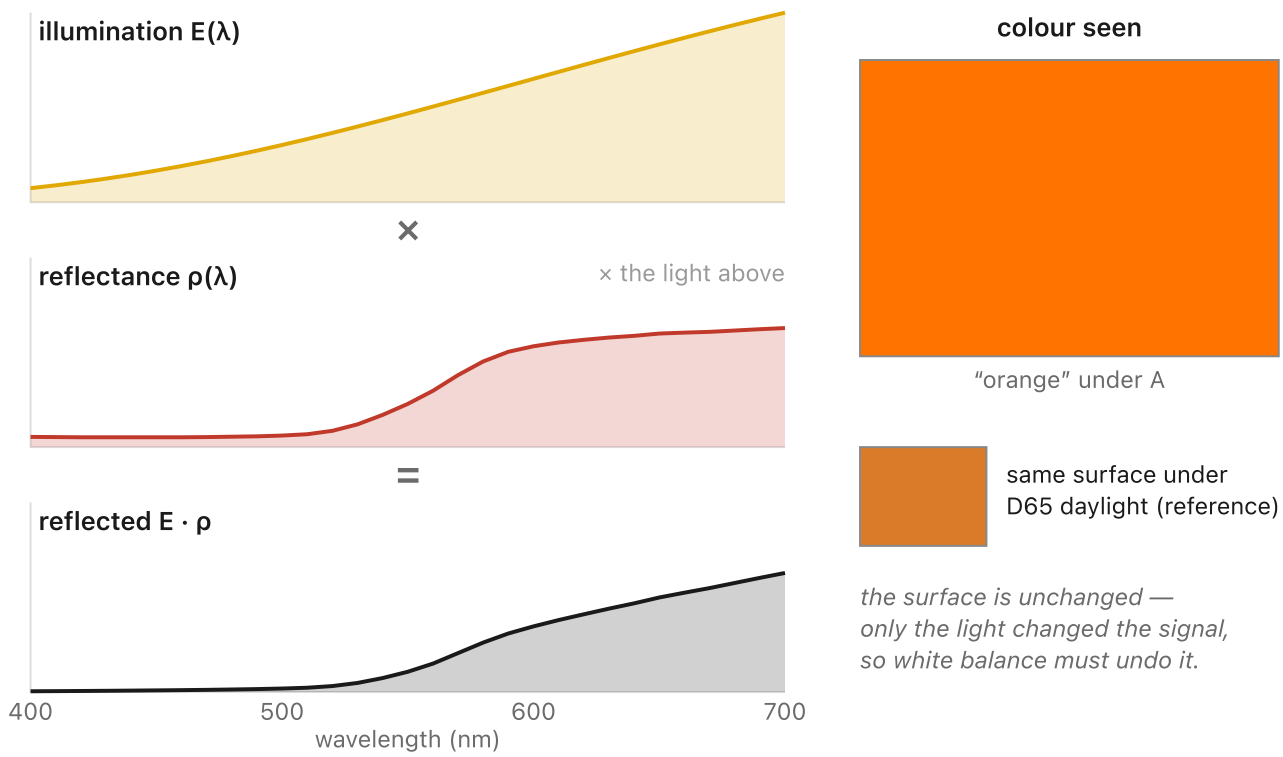


Figure 2.1.28 (reproduced). *Color is illumination × reflectance. A surface's fixed reflectance spectrum ρ(λ) (here an apple's, high in the reds) is multiplied per wavelength by the illuminant spectrum S(λ) actually present. The product C(λ) = S(λ)·ρ(λ) is the light that reaches the eye. Change the illuminant and the product changes even though the surface has not: the same reflectance under white versus reddish versus greenish light yields three different stimuli. One swatch shows the surface's unadapted appearance under the chosen illuminant; a smaller reference swatch shows it under D65 for comparison.*

💡 **BIG LESSON (L3.1) — A CONE RESPONSE IS A DOT PRODUCT OF THE SPECTRUM WITH THE CONE'S SENSITIVITY**

Each cone excitation is an integral of spectral density times the appropriate sensitivity. A sampled implementation must include wavelength-bin widths (or absorb them into the matrix). Three excitations summarize photopic color matching for a fixed observer and viewing condition; they do not recover the spectrum.

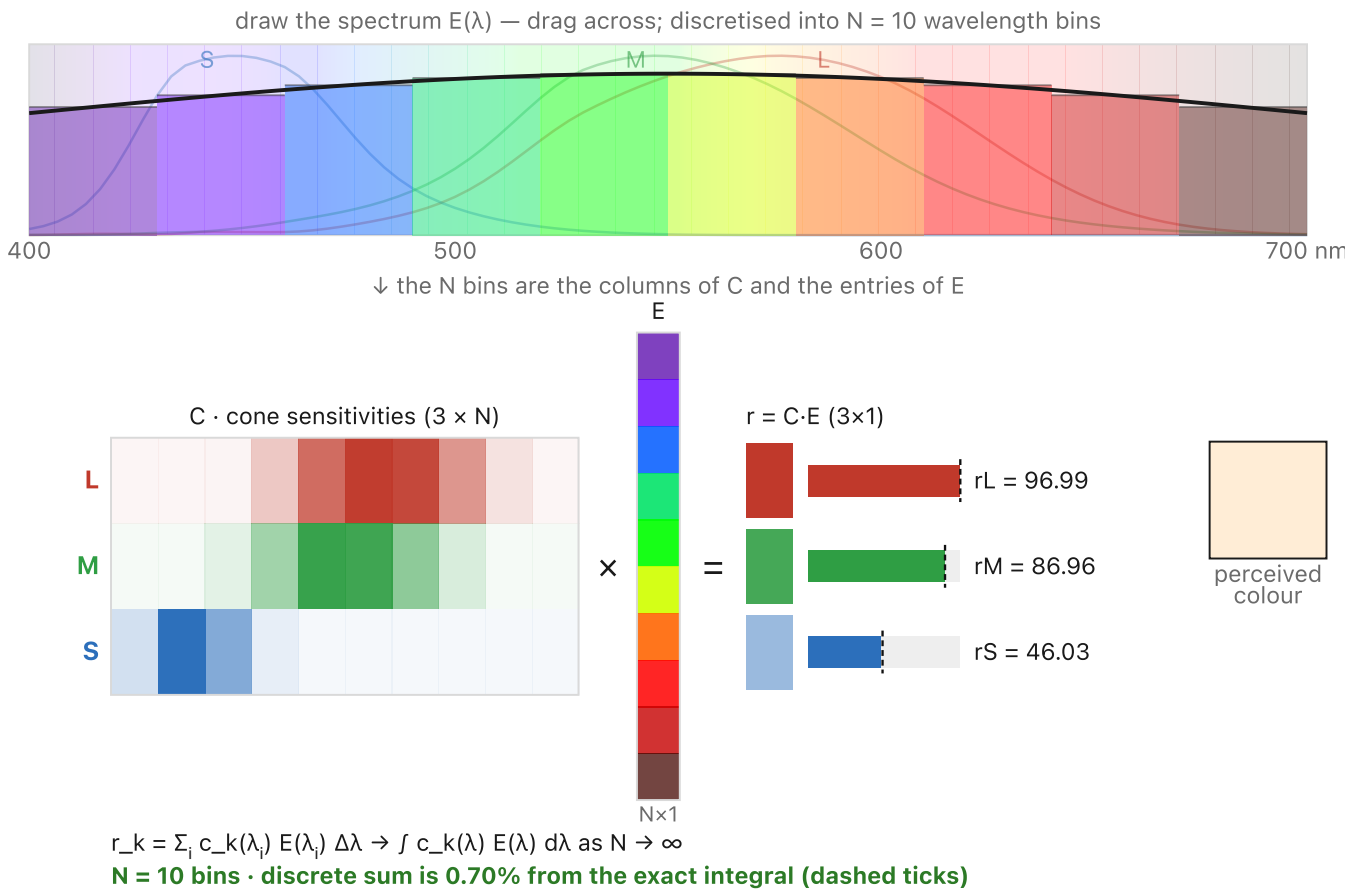


Figure 3.2.5 (reproduced). *Cone response as a matrix–vector product* $\mathbf{r} = \mathbf{C}\,\mathbf{E}$. *The spectrum is discretized into* N *samples (the vector* $\mathbf{E}$*); the three cone sensitivities are the rows of the* $3 \times N$ *matrix* $\mathbf{C}$*; the product is the three cone responses. In the limit* $N \to \infty$ *the spectrum is infinite-dimensional and the sum becomes an integral. Interactive: draw the spectrum by dragging across the plot, and move the bins slider, and as N grows the staircase sum (a Riemann sum) hugs the continuous curve.*

💡 BIG LESSON (L3.2) — METAMERS: DIFFERENT SPECTRA, SAME CONE RESPONSE

Different spectra can yield the same cone excitations for a specified observer. This enables color reproduction without reproducing the spectrum, but a fixed display can match only colors within its gamut. Matches between reflecting materials can fail when the illuminant changes.

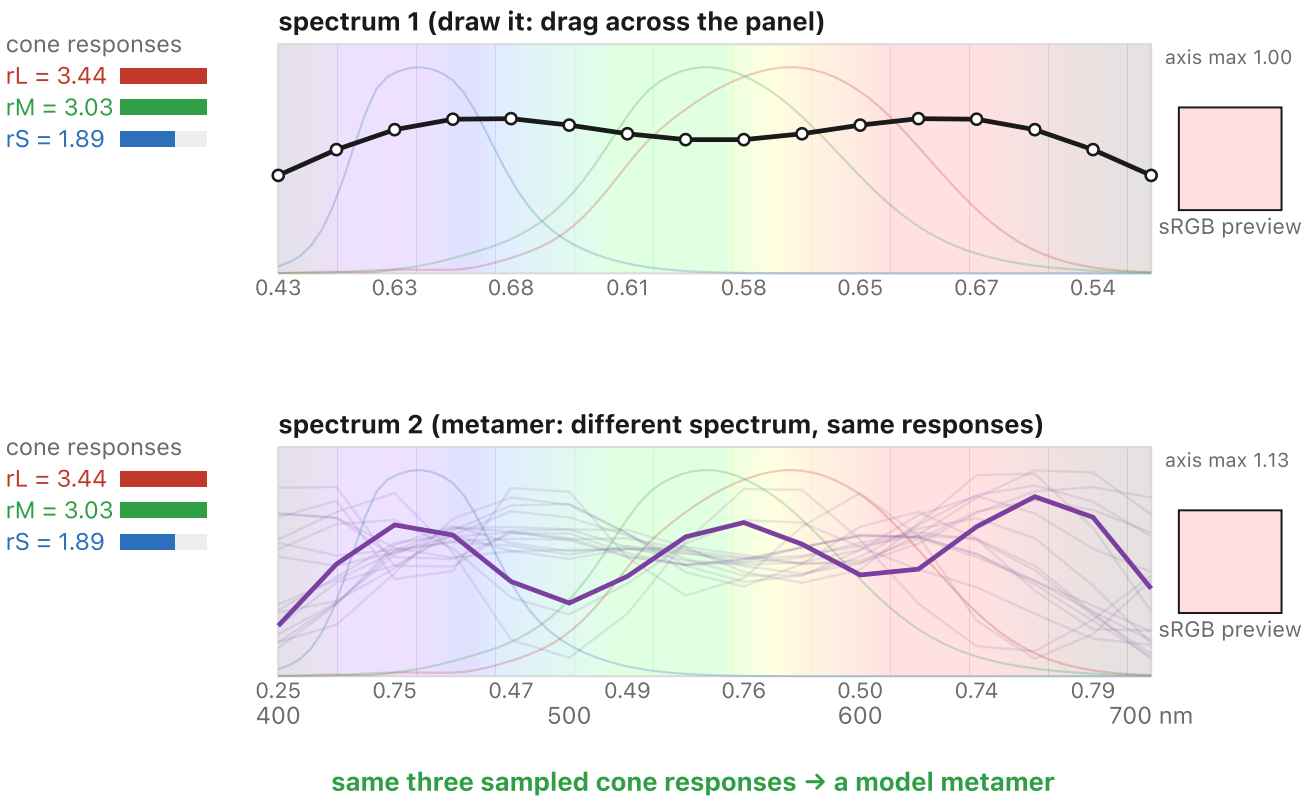


Figure 3.2.10 (reproduced). *Metamerism, made constructive. The top spectrum is projected onto the three cone curves to give the responses* (r_L, r_M, r_S)*; the bottom spectrum is built to be different at every wavelength yet share the same three responses, so both collapse to the same triple and the same perceived color. Generate a fresh metamer, switch families (smooth, spiky with a chosen number of spikes, or block/step), or overlay many family members, all sharing identical cone responses. Many spectra, one response.* Interactive: draw the top spectrum by dragging across the panel; the bottom spectrum is generated to be different at every wavelength yet share the same three cone responses.

💡 **BIG LESSON (L3.3) — METAMERISM CAN BREAK WHEN THE ILLUMINANT CHANGES**

A metameric match is a match between two **products** of reflectance and illuminant (**L2.6** show). Swap the illuminant and you multiply by a different spectral shape, so two surfaces that looked identical under one light can **diverge under another**: *illuminant metamerism*. It is the headache behind prints that shift between the proofing booth and the living room, and fabrics that match in the shop but clash in daylight; color-critical work must always name the light a match is judged under.

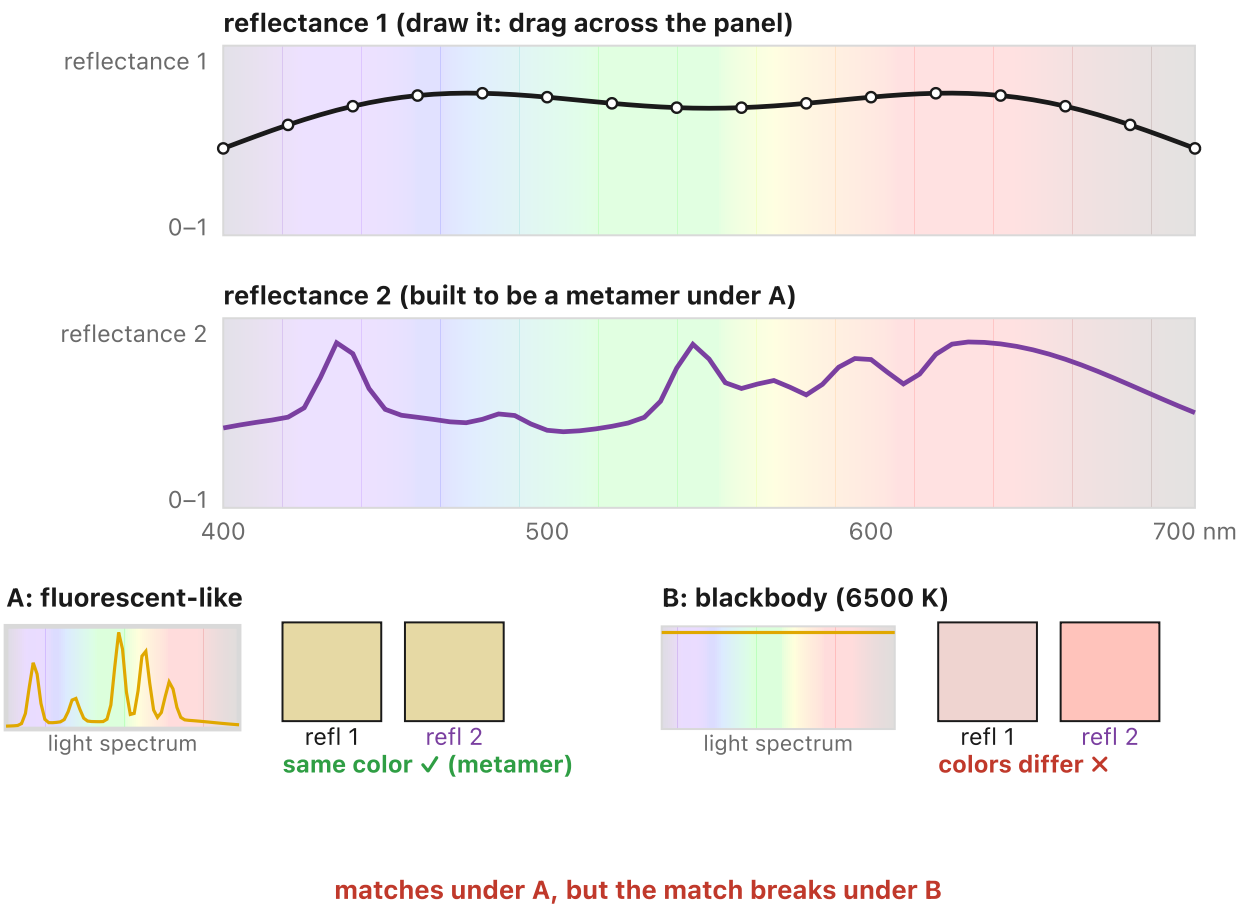


Figure 3.2.13 (reproduced). *Metamerism depends on the light (interactive). Two **reflectance** spectra are built to be metamers under one illuminant, say fluorescent: multiplied by that light they land on the same three cone responses, so they look identical. Under a second illuminant, say daylight, the same two reflectances multiply by a different spectral shape and their colors pull apart. Draw your own reflectance and press "new metamer" for a fresh partner that matches under illuminant A; the two swatch pairs show the match holding under A and breaking under B. Press "new random metamer" for an ordinary partner, or "new adversarial metamer" for a strong mismatch, a partner chosen among the metamers of A by a projected power iteration to change the response under B as much as it can find; "new adversarial reflectance 1" goes further and moves reflectance 1 itself toward the edges of the physical [0,1] range, so its partner can diverge further. Both search for a large mismatch; neither is a proved maximum under the [0,1] constraint. The "match under both" checkbox does the opposite, much harder job: find a reflectance that matches under **both** lights at once, a perturbation that lives in the null space of both illuminants. That extra constraint is exactly the challenge of printing, where an ink set mixed to match a target under the proofing light drifts under the customer's bulbs, and matching everywhere costs many more degrees of freedom.*

💡 BIG LESSON (L3.4) — COLOR VISION IS MULTI-STAGE: TRICHROMATIC, THEN OPPONENT

Cone excitations are followed by neural comparisons and adaptation. Sum-and-difference models illustrate this processing, but their axes are not exact perceptual hue axes. Engineering luma/chroma transforms exploit related sensitivity differences without reproducing the neural pathway.

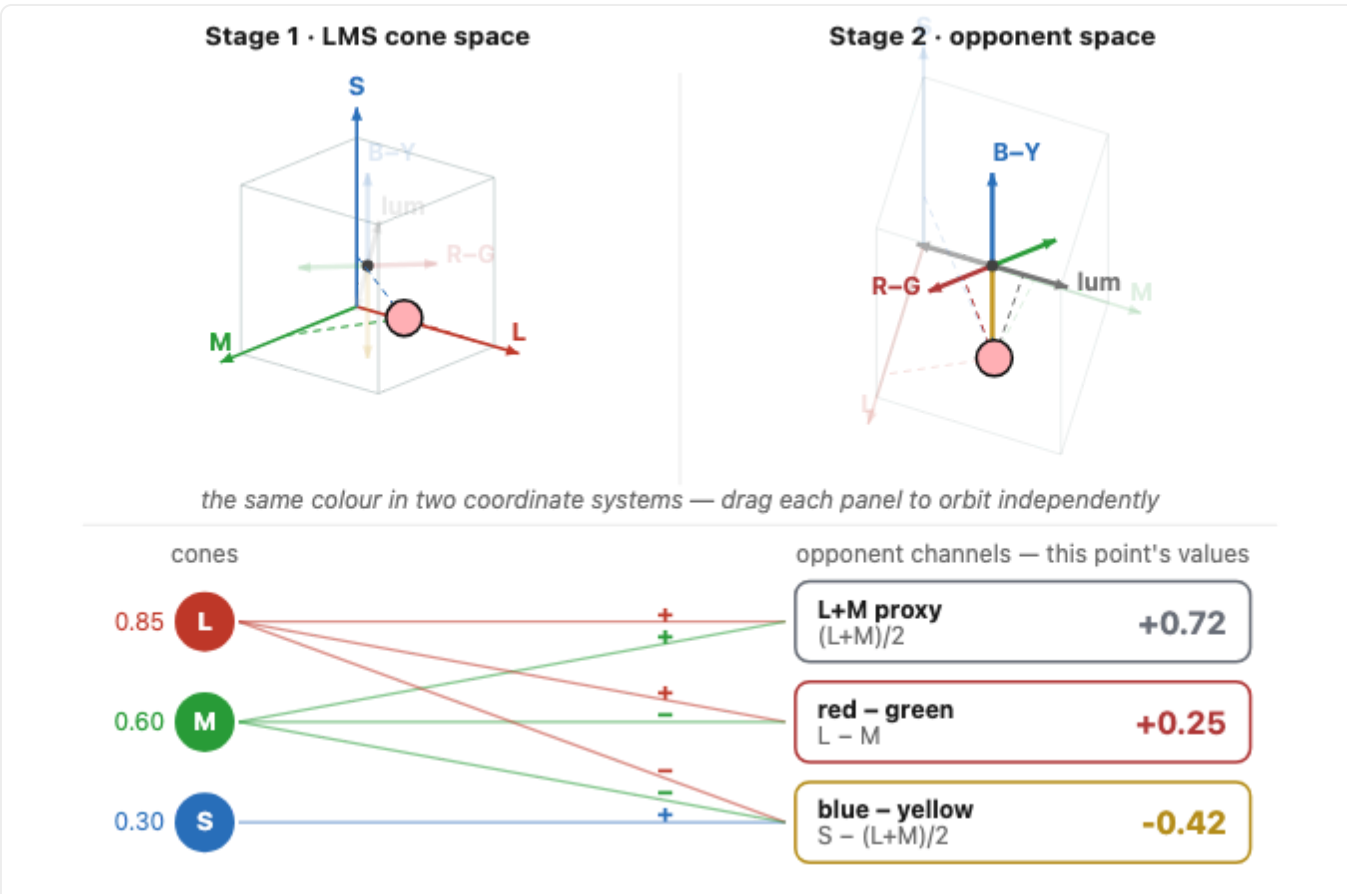


Figure 3.2.19 (reproduced). *Human color in two stages, shown as geometry. Stage 1 is the three cone responses (L, M, S), a point set with the sliders and held inside the physically realizable gamut (inputs outside it are projected to its boundary). Stage 2 recodes that point onto opponent axes, a luminance proxy (L+M)/2, red-green (L–M), and blue-yellow (S–(L+M)/2). The same point is shown in two views side by side, the LMS cone space and the opponent space, where the geometry is sheared so the opponent axes are orthogonal; each orbits on its own. The equal weights are a schematic opponent model, not a calibrated luminance or DKL transform. The opponent channels are differences of cone signals, not raw cones, which is why there is no reddish-green, why an afterimage rebounds the opposite way, and why luma/chroma encodings compress: luminance carries the detail and the two chroma differences are smooth. Drag either view to orbit it; the swatch shows the point's color (LMS→sRGB). Below the two panels a live wiring diagram, in the same style as the opponent-channels schematic above, carries the three cone nodes L, M, and S (each showing the point's cone value) through their + and – connections into the three opponent boxes, which print each channel's formula and its value for this point: luminance (L+M)/2, red–green L–M, and blue–yellow S–(L+M)/2, colored toward red or green and blue or yellow by sign.*

💡 **BIG LESSON (L3.5) — PRIMARY COLORS ARE A CHOICE, NOT A FACT ABOUT LIGHT**

Specify the purpose when naming primaries: sources whose light is added, inks or filters that absorb light, or perceptual hue judgments. RGB and CMY describe common engineering choices, not unique sets required by nature. The chosen spectra and mixing process determine what can be reproduced.

💡 **BIG LESSON (L3.6) — HUMAN COLOR IS NON-ORTHOGONAL *AND* NON-NEGATIVE**

Cone sensitivity curves overlap, and physical spectra and emitted primary powers are nonnegative. Cone responses are measurements, not spectrum coefficients. Matching a response triple with three primary lights requires solving for their powers; a negative solution identifies a color outside those primaries' additive gamut.

💡 **BIG LESSON (L4.2) — COLOR ANALYSIS AND SYNTHESIS REQUIRE DIFFERENT SPECTRAL DISTRIBUTIONS**

Sensitivity curves measure a spectrum; emission primaries construct one. Reusing the sensitivity curves as primaries and the measured responses as weights generally gives the wrong color. Calibrate how each primary stimulates the observer, then solve for its weight.

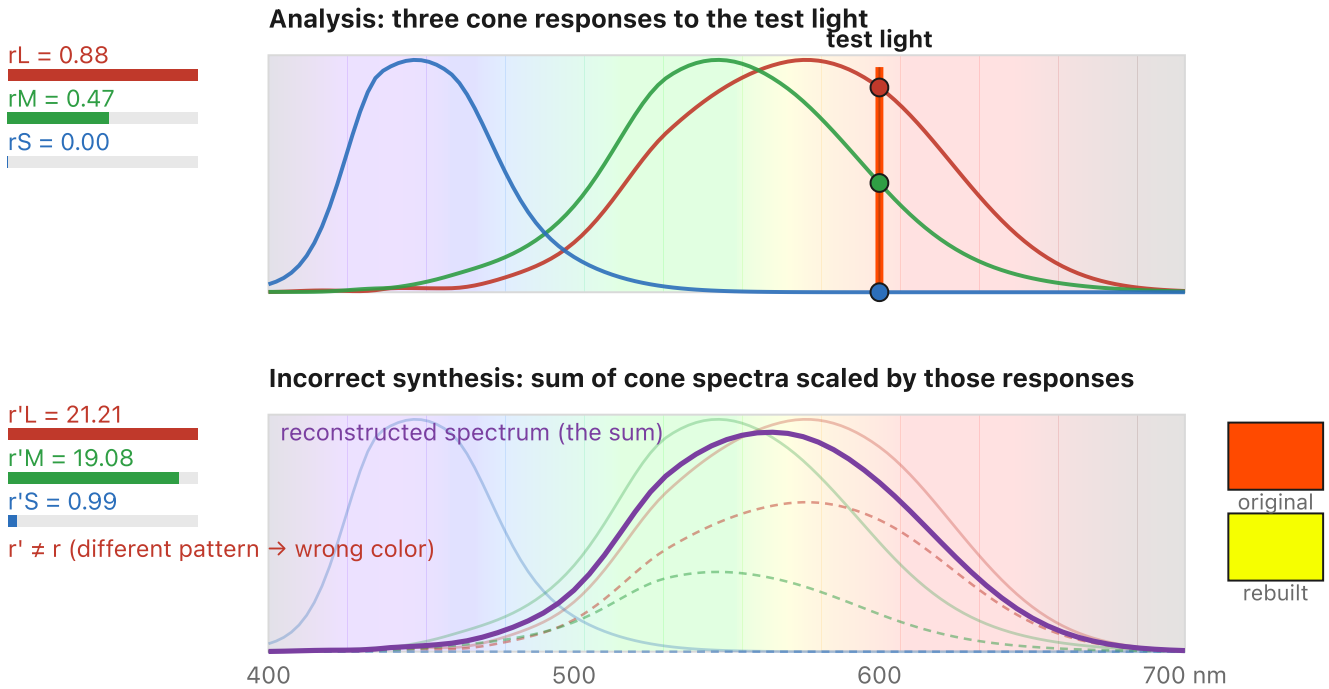


Figure 4.1.3 (reproduced). *Why the cone curves are the wrong synthesis primaries (interactive). Pick a monochromatic test light with the slider. Top (analysis): the three cone responses are the heights of the L, M, S curves at that wavelength. Bottom (incorrect synthesis): rebuild the light by scaling each cone spectrum by its own response and summing them. The sum is a broad, washed-out spectrum, and when it is measured back through the cones its responses are not the originals (they are the original responses multiplied by the cone overlap matrix), so its perceived color is wrong. Reusing the analysis basis for synthesis fails, which is why real display primaries are not the cone sensitivities.*

💡 **BIG LESSON (L4.3) — LINEAR TRISTIMULUS COORDINATES ARE A MATRIX APART**

XYZ, a compatible LMS system, and linear RGB are different coordinates for a fixed observer's three-dimensional color matches. Nonlinear RGB encodings and perceptual spaces add specified nonlinear transforms. A real camera's raw measurements usually require an approximate color correction; an appearance model also needs viewing conditions.

💡 BIG LESSON (L4.8) — THERE IS NO SINGLE RGB: GAMUT VERSUS BIT BUDGET

Choose a working gamut large enough for the content and operations, with sufficient numeric precision. Choose a delivery space the receiving system can interpret. Wider primaries alone do not improve an image, and converting an sRGB image to a wider space does not recover colors previously clipped.

💡 BIG LESSON (L4.12) — ADDITIVE PRIMARY CHROMATICITIES FORM A CONVEX HULL

Nonnegative mixtures of fixed additive primaries lie within their chromaticity hull. Three primaries give a triangle. Maximum drive and black level also constrain the full three-dimensional gamut. Subtractive printing is not described by the same triangle rule: ink interactions and the paper must be characterized.